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Service Paper
AN EXPERIMENT IN THE TEACHING OF PLANE GEOMETRY
COMPARING
THE FORMAL METHOD WITH THE INFORMAL METHOD

Submitted by
John William Jacobs
(B.S. in E.E., Northeastern University, 1932)
(B.S. in Ed., Bridgewater State Teachers College, 1937)

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

1949

School of Education

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31508

First Reader: Henry W. Syer, Assistant Professor of Education

Second Reader: Donald D. Durrell, Professor of Education

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
Introduction	
Problem.....	VI
Method of Solution.....	VII
I Previous Investigation in the Field.....	1
Search for Research Papers.....	1
Inter-Library Loan.....	2
Authors of Textbooks.....	3
Why Postulate.....	5
What Postulate.....	7
Opposition.....	10
How Much Formal Proof.....	15
II Experimental Procedures.....	17
Groups Selected.....	17
School Background.....	17
Equating Data, List of Tables.....	18
Pairing.....	19
Matching Groups.....	20
Summary of Equivalence Factors, Table I.....	22
Theorems to Postulate.....	22
Types of Textbooks.....	24
Timetable of Work.....	25
Achievement Tests Used.....	27
Reviews of Tests.....	28
Achievement Data, List of Tables.....	30
III General Summary.....	31
Comparison of Groups.....	31
Equating of Groups.....	32
Achievement of Groups.....	34
Summary of Differences and Sigma _{Ds} , Table II.....	36
Gain in Achievement, Table III.....	38
Classification of Textbooks.....	39



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TABLE OF CONTENTS (continued)

<u>Chapter</u>	<u>Page</u>
IV Conclusions and Recommendations.....	40
Difference between Methods.....	40
Time Required.....	40
Whole and Partial Groups.....	40
Discussion of Conclusions.....	41
Recommendations.....	42
Appendix A.....	44
Appendix B.....	73
Bibliography.....	89

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LIST OF TABLES

Appendix A

<u>Number</u>		<u>Page</u>
1.	A Comparison of Axioms and Postulates; Theorems, Corollaries, and Constructions of six often-used Textbooks.....	44
2.	Suggested Places in Plane Geometry in which we can Shorten, Eliminate, or Postulate to Allow Time to Introduce Space Concepts.....	45
3.	Chart Showing Pairs of 1948 and 1949 Plane Geometry Students, Plotting Intelligence Quotients versus Average of Mathematics Marks.....	47
4.	Significance of the Difference Between the Means - Unequated Groups.....	48
5.	Significance of the Difference Between the Means - Equated Groups.....	49
6.	Theorems Postulated from Welchons & Krickenberg <u>Plane Geometry</u>	50
7.	Classification of Plane Geometry Textbooks	52
8.	Timetable of Work - 1948 Group vs. 1949 Group.....	55
9.	Semi-Final and Final Test Scores for 1948 and 1949 Groups.....	56
10.	Conversion of 1948 Semi-Final Scores to Sigma Scale and T-Scores.....	58
11.	Conversion of 1949 Semi-Final Scores to Sigma Scale and T-Scores.....	60
12.	Conversion of 1948 Final Scores to Sigma Scale and T-Scores.....	62
13.	Conversion of 1949 Final Scores to Sigma Scale and T-Scores.....	64

CHAPTER 10
THEORY OF THE EARTH

1. The Earth is a sphere of radius 6370 km.
2. The Earth is composed of three main layers: the crust, the mantle, and the core.
3. The crust is the outermost layer, with a thickness of about 10 km.
4. The mantle is the layer below the crust, extending to a depth of about 2900 km.
5. The core is the innermost layer, extending to the center of the Earth.
6. The core is divided into two parts: the outer core and the inner core.
7. The outer core is a liquid layer, while the inner core is a solid sphere.
8. The temperature increases with depth in the Earth.
9. The pressure also increases with depth in the Earth.
10. The density of the Earth increases with depth.
11. The Earth's magnetic field is generated by the outer core.
12. The Earth's rotation causes the equatorial bulge.
13. The Earth's shape is an oblate spheroid.
14. The Earth's mass is approximately 5.97×10^{24} kg.
15. The Earth's volume is approximately 1.08×10^{21} m³.
16. The Earth's average density is approximately 5515 kg/m³.
17. The Earth's surface area is approximately 5.1×10^{14} m².
18. The Earth's circumference is approximately 40,075 km.
19. The Earth's radius is approximately 6370 km.
20. The Earth's diameter is approximately 12,742 km.

LIST OF TABLES (continued)

<u>Number</u>		<u>Page</u>
14.	Significance of the Differences between the Means of the 1948 and 1949 Groups on the Semi-Final and Final Test Scores.....	66
15.	Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the Groups on Semi- Final Tests.....	67
16.	Comparison of Scores and Significance of the Difference between the Means for the Lower Halves of the Groups on Semi-Final Tests.....	68
17.	Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the Groups on Final Tests.....	69
18.	Comparison of Scores and Significance of the Difference between the Means for the Lower Halves of the Groups on Final Tests.....	70
19.	Graphs of Achievement on Semi-Final Tests for 1948 and 1949 Groups.....	71
20.	Graphs of Achievement on Final Tests for 1948 and 1949 Groups.....	72

— 244 —

[Return to top](#)

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LIST OF TABLES (continued)

Appendix B

<u>Number</u>		<u>Page</u>
1.	Graphs of Intelligence Quotient versus Frequency of All Eleventh-Grade Students and of Plane Geometry Students in 1948.....	73
2.	Graphs of Intelligence Quotient versus Frequency of All Eleventh-Grade Students and of Plane Geometry Students in 1949.....	74
3.	Data Used in Calculating Mean Intelligence Quotient and Sigma of Unequated Groups..	75
4.	Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups - First Trial.....	77
5.	Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups....	79
6.	Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1948 Group..	81
7.	Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1949 Group..	83
8.	Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1948 Group.....	85
9.	Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1949 Group.....	87

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INTRODUCTION

Do students of plane geometry show a greater achievement as measured by standardized tests when taught by the formal method or when taught by the informal method?

For the purpose of this paper, the terms formal method and informal method will have to be carefully defined. When the idea for this paper was first presented to the seminar group, the title read as follows: An Experiment in Teaching Plane Geometry Using the Traditional Method versus the Postulational Method. The traditional method was defined as consisting of units of work where the formal proof of theorems was required before they could be used in proofs of original exercises. The postulational method was defined as assuming or postulating the proof of theorems required to prove the original exercises of the unit.

After much discussion, it was decided that less confusion would result if the terms traditional method and postulational method were changed to formal method and informal method. Formal method will be defined as the method of teaching plane geometry by requiring the learning or committing to memory of formal proofs of theorems for reproduction before attempting to use them in

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the solution of original exercises. In the informal method the proofs of certain basic theorems which are essential to the proof of other theorems and the nature of their proof is such that an artificial method such as superposition or indirect proof is required will be postulated. Understanding of the theorems will be gained by intuitive, informative, experimental, or inductive teaching. The use of all of the terms is practically synonymous, and the authorities in the field recommend that the word "informal" be used.

Comparison of the results obtained by two groups of plane geometry students of approximately equal ability taught by the two methods defined in the previous paragraphs should tend to influence the type of course to be given and perhaps the textbook to be used.

This study will be set up as an experiment with two groups equated as to mental maturity by the California Short-Form Test of Mental Maturity.^{1/}

The two groups were tested halfway through the course and again at the end of the course. The gain or loss between half-year achievement and the final achievement for each group were compared to see if any significant difference could be determined.

^{1/}Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, California Short-Form Test of Mental Maturity, Advanced S-Form, (California Test Bureau, Los Angeles, 1939)

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CHAPTER I

PREVIOUS INVESTIGATION IN THE FIELD

There appear to be no readily available studies which attempt to prove experimentally that a theorem a day is not the way to teach plane geometry.

In arriving at this conclusion, the writer first examined the card catalogue files of theses in the Boston University School of Education Library. This examination revealed how few theses in the field of mathematics and particularly in plane geometry were available. It was my good fortune, however, to discover that my seminar adviser, Professor Henry W. Syer, had recently compiled an extensive list of theses in the field of mathematics which are to be found in the libraries of schools and colleges all over the United States. This list covered a period of about twenty years. Working over this list, I compiled a sub-list of thirty theses pertaining to plane geometry. About half of this group were found to be studies comparing the directed study or laboratory method of teaching with the traditional lesson-learning plan. Of the remaining fifteen, six dealt with motivating materials in plane geometry. It was not possible to tell from the titles in every instance what the remaining studies actually were reporting on.

THE HISTORY OF

THE UNITED STATES OF AMERICA

FROM THE FIRST SETTLEMENTS TO THE PRESENT TIME

BY
J. W. FULTON, ESQ.,
OF NEW-YORK.

NEW-YORK:
PUBLISHED BY
J. W. FULTON, 101 NASSAU ST.
1854.

Through the courtesy of the Boston University School of Education Library and the Inter-University Library Loan Service, several of the theses with the doubtful titles and several which were thought might apply were requested for examination. In every instance, however, the theses examined were found not to have any direct bearing on the work this paper is experimenting with.

The studies examined brought to light some experimental procedures, however, in the handling of conclusions which the writer thought might apply to this study.

There are numerous indications in recent geometry textbooks that much thought has been given to the subject of postulation. In addition to the opinion of textbook writers, many articles by teachers of mathematics were found which indicate that the subject has been discussed pro and con. In the investigation of the literature of the field which follows, an attempt will be made to indicate what textbook writers are doing. Second, some of the arguments both for and against postulation will be given which were located in professional periodicals and books.

It has been interesting to note how the postulation idea has developed from the idea of postulating only the congruence theorems as expressed by Shibli

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to that of postulating theorems on congruence, parallelism, similarity, and inequalities as indicated by Reeve, Birkhoff and Beatley, and others.

The quotations which follow were chosen for three main reasons: First, they discussed what should be postulated; second, why postulate; third, they recorded opposition to the idea. The first is taken care of by the articles which follow by Reeve, Shibli, Christofferson, and Welkowitz. The second by the quotes from Mensenkamp, Fawcett, Purdue Workshop, and Nygaard. The third, the recorded opposition, as evidenced by articles by Royall, Salkind, Barber, and Lynch.

Herberg and Orleans^{1/} in their A New Geometry for Secondary Schools state "By treating as assumptions the conditions for congruence, parallelism, and similarity, a thoroughly logical course has been built that is considerably simpler than the usual course."

In a similar vein, Crawford and Schnell^{2/} in Clear Thinking, An Approach through Plane Geometry offer as one of their twelve basic principles, "No. 2, Formal geometry progresses more rapidly and with more understanding when

^{1/} Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston; D.C. Heath and Company, 1948), Preface p. 3

^{2/} Leroy H. Schnell and Mildred Crawford, Clear Thinking, An Approach through Plane Geometry, (New York; Harper & Brothers, 1943), Preface p. 10

1871
The first of the year was a very dry one, and the
season was generally unfavorable for the crops.
The wheat was very small, and the corn was
very poor. The cotton was also very small, and
the sugar cane was very poor. The rice was
very small, and the other crops were very poor.
The weather was very dry, and the crops were
very small. The season was generally unfavorable
for the crops. The wheat was very small, and
the corn was very poor. The cotton was also
very small, and the sugar cane was very poor.
The rice was very small, and the other crops
were very poor. The weather was very dry, and
the crops were very small. The season was
generally unfavorable for the crops.

The second of the year was a very wet one, and
the season was generally favorable for the crops.
The wheat was very large, and the corn was
very good. The cotton was also very large, and
the sugar cane was very good. The rice was
very large, and the other crops were very good.
The weather was very wet, and the crops were
very large. The season was generally favorable
for the crops. The wheat was very large, and
the corn was very good. The cotton was also
very large, and the sugar cane was very good.
The rice was very large, and the other crops
were very good. The weather was very wet, and
the crops were very large. The season was
generally favorable for the crops.

The third of the year was a very dry one, and
the season was generally unfavorable for the crops.
The wheat was very small, and the corn was
very poor. The cotton was also very small, and
the sugar cane was very poor. The rice was
very small, and the other crops were very poor.
The weather was very dry, and the crops were
very small. The season was generally unfavorable
for the crops. The wheat was very small, and
the corn was very poor. The cotton was also
very small, and the sugar cane was very poor.
The rice was very small, and the other crops
were very poor. The weather was very dry, and
the crops were very small. The season was
generally unfavorable for the crops.

extreme care is taken in developing fundamental concepts. No good or lasting purpose is served by rushing into a study of formal demonstrative proofs."

Birkhoff and Beatley^{1/} in Basic Geometry. "The traditional approach to demonstrative geometry involves careful study of certain theorems which the beginner is eager to accept without proof and which he might properly be led to take for granted as assumption or postulate. Such an approach obscures at the very outset the meaning of 'proof' and 'demonstration'. The employment of superposition in the proof of some of these theorems is even more demoralizing. This method of proof is so out of harmony with the larger aims of geometry instruction that despite the validity, its use is commonly restricted to those few cases for which no better method can be found."

Reichgott and Spiller^{2/} in Today's Geometry. "Formal demonstration is kept at a minimum. No attempt is made to adhere to a rigorous proof. There are more postulates, assumptions, and assumed theorems than in traditional geometry. Numerous exercises based on these theorems and postulates afford practice in logical development."

^{1/} George D. Birkhoff and Ralph Beatley, Basic Geometry, (Scott, Foresman and Company, 1941, Chicago), Preface p. 3

^{2/} David Reichgott and Lee R. Spiller, Today's Geometry, (New York, Prentice-Hall, Inc., 1938), Preface p. 7

Let us next consider what teachers in the field have to say on the subject. Reeve^{1/} in The Teaching of Geometry gives the following list of particularly important basic theorems:

1. The congruence theorems.
2. The equality of alternate angles in case of parallels.
3. The sum of the angles of a triangle.
4. The theorems on similar figures.
5. The Pythagorean Theorem.
6. The measurement of angle between two chords.

He suggests that we might well postulate all but the third and could do most originals. "So important are these few propositions that if we had no others with which to work we could with these alone prove a large proportion of the original exercises of plane geometry." and he continues, "Proof of propositions in textbooks should be models or should not be proved at all. In fact we could easily omit the proofs of many conventional propositions and the pupils would gain in every respect by such omission. Eg. Proof for each of the family of parallelogram propositions usually begins with 'Diagonal of a parallelogram divides

^{1/} National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University,) p 1-28

(1) The first of the three main principles of the
constitution is that the state is a sovereign
entity, independent of any external power.

(2) The second principle is that the state is a
unitary entity, meaning that there is no
division of power between different levels of government.

(3) The third principle is that the state is a
democratic entity, meaning that the power of the
state is derived from the people.

(4) The fourth principle is that the state is a
constitutional entity, meaning that the power of the
state is limited by the constitution.

(5) The fifth principle is that the state is a
legal entity, meaning that the state is bound by
the law.

(6) The sixth principle is that the state is a
moral entity, meaning that the state has a
duty to its citizens.

it into two congruent triangles.' This and all such simple propositions should be treated as original exercises."

Birkhoff and Beatley^{1/} in A New Approach to Elementary Geometry present the following argument, "What is the point in telling beginners that we shall assume certain 'self-evident truths', and then asking them to prove certain other propositions which they regard as equally self-evident truths? Would they not come to a greater understanding of the nature of a proof through the effort to prove easy 'originals' which are not too plausible and which seem therefore to require justification?"

Mensenkamp^{2/} in Some Desirable Characteristics in a Modern Plane Geometry Text presents these factors to be considered: "First, there is the pupil himself. The number enrolled constitutes a much larger proportion of the total population than it did a decade or so ago. This means there are now present in our tenth grade classes many more students representing the lower levels of mental ability than was formerly the case. Most students of this type do not intend to go to college and it is hard to interest them in a difficult subject like

1/ National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University), p 86-96

2/ Ibid., p 199-206

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geometry, especially if the textbook presentation is of an abstract or formal character. The two reports which have given direction and sanction to geometry reform in this country during the past several years are: The re-organization of mathematics in secondary schools, a report by the National Committee on Mathematics Requirements (1923), and a report of the College Entrance Examination Board on Geometry requirements."

Fawcett^{1/} in The Nature of Proof states "Actual classroom practice indicates that major emphasis is placed on a body of theorems to be learned rather than on the method by which these theorems are established. Pupil feels theorems are important in themselves and in his earnest effort to know them resorts to memorization."

Shibli^{2/} in Recent Developments in the Teaching of Plane Geometry. "Some teachers advocate postulating congruence by sides along with the other two congruence theorems in the interests of simplicity and consistency. Some fear that the movement toward free postulation may go too far."

^{1/} National Council of Teachers of Mathematics, The Nature of Proof, Thirteenth Yearbook of National Council of Teachers of Mathematics (New York; Bureau of publications of Teachers College, Columbia University), p. 117

^{2/} J. Shibli, Recent Developments in the Teaching of Plane Geometry, (1932, Penn State College, published by J. Shibli), p. 104

The first of these is the fact that the
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The second of these is the fact that the
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The third of these is the fact that the
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Christofferson^{1/} in Geometry Professionalized for Teachers suggests the postulation of congruent triangles rather than proof by superposition. "The chief defense for complete postulation of all three theorems, in addition to the abandonment of superposition is simplicity and understanding at the beginning of the course."

He goes on to say that the number of Fundamental Theorems chosen should be based upon whether they are to be used again in the proof of other theorems and mentions the possibility of using only ten constructions and twenty theorems or a total of thirty as compared to the average of 195 constructions, theorems, and corollaries in six often-used textbooks. See Appendix A, Table 1 for table showing this comparison taken from Christofferson's study.

It is interesting to note that the purpose of this study was to discover how few really fundamental theorems are needed upon which to build the entire structure of geometry. This was from a professional and not a mathematical point of view.

^{1/} H.C.Christofferson, Geometry Professionalized for Teachers, (George Banta Publishing Company, Menasha, Wisconsin, 1933), p. 34-37

9

The following extracts from the list of "suggested places in plane geometry in which we can shorten, eliminate, or postulate to allow time to introduce space concepts" were taken from the report of the committee^{1/} working on the first six weeks in plane geometry of the second annual Purdue Mathematics Workshop. These extracts emphasize a point this paper is trying to determine -- whether or not too much time is being given to formal proof.

1. It is suggested that we postulate the theorem "one and only one line can be drawn from (at) a given point perpendicular to a given line."
2. If the hypotenuse - angle congruency proposition is proved by superposition, it might be postulated.
3. It might be possible to postulate the theorem "If two angles have their sides respectively perpendicular, they are either equal or supplementary."
4. The theorem "The sum of the angles of a polygon of n sides is $(n-2)$ straight angles" may be postulated or proved informally.
5. In connection with inequalities in a triangle or a circle, certain theorems and corollaries might be postulated.
6. The concurrency propositions, having been intuitively established in the introduction, may be postulated.
7. Postulate the theorem "If two polygons are similar, they can be divided into triangles which are similar and similarly placed" and its converse.
8. Postulate the continuity of the Pythagorean Theorem as applied to similar polygons constructed upon the three sides of a right triangle.

A complete list will be found in Appendix A, Table 2.

^{1/} Committee on the First Six Weeks in Plane Geometry, Second Annual Purdue Mathematics Workshop, June 16-June 28, 1947

Following along the same line, Nygaard^{1/} in A Functional Revision of Plane Geometry says, "The writer has made some effort to determine what material, usually included in plane geometry, is of little or no value in later mathematics or science courses. He is convinced that a number of the theorems dealing with the circle have no future use -- for instance, the measurement of all sorts of angles in terms of their intercepted arcs." "Theorems based on dividing a line segment externally are in the same class." "Comprehensive proofs of the theorems dealing with the area of rectangles, parallelograms, triangles, and trapezoids would come under the same ban." "Many of the relationships involved in triangles, parallelograms, and circles could be more efficiently presented as lists of characteristic properties or as student exercises than as theorems completely proved in the textbook."

Readers who are interested may wish to read A Reply to Mr. Nygaard^{2/} by Norman N. Royall, Jr., Winthrop College, Rock Hill, South Carolina, or a further

^{1/} P.H. Nygaard, A Functional Revision of Plane Geometry, The Mathematics Teacher (October 1941), Vol. 34, No. 6, pp 269-273

^{2/} Norman N. Royall, Jr., A Reply to Mr. Nygaard, The Mathematics Teacher (April 1942), Vol. 35, No. 4, pp 179-181

discussion of Mr. Nygaard's article entitled The War on Euclid, by Charles Salkind 1/.

Royall is concerned with the harm that may be done to sound instruction in mathematics by the type of discussion quoted above. He suggests that when Mr. Nygaard or anyone else makes "A Functional Revision of Plane Geometry" one cannot be sure that what is left is plane geometry, and he points out that one of the primary objectives of a plane geometry course is to give the students a chance to learn the nature of a deductive proof.

Salkind agrees with several of the premises, but feels that intuitive geometry is taken care of by our Junior High Schools. He says, "However, whether this type of geometry teaching, call it intuitive or informational or experimental or inductive, precedes the unit of demonstrative geometry or is taught simultaneously with it, it is imperative for us, as purveyors of Mathematical Knowledge, to know the nature of demonstrative geometry."

1/ Charles Salkind, The War on Euclid, The Mathematics Teacher (May 1942), Volume 35, No. 5, pp 205-207

Harry C. Barber^{1/} in Random Notes on Geometry Teaching says "There has been much discussion of what should be done with superposition proofs. The plan often advocated is to postulate all the theorems usually so proved. In England the reason given is that when we move a figure we cannot be certain that it does not change in size and shape. In the United States the reasons given are that superposition is not a suitable method at the outset; it is not readily understood by the beginner; and it interferes with the later use of other methods of proof." Barber goes on to point out that all direct measurement is a process of superposition. He says, "To omit the superposition proofs at the beginning of plane geometry is to miss the strongest link we have between everyday experience and the argument of geometry. Here is a place where teachers need to do battle against a current unfortunate trend."

Lynch^{2/} says, "outstanding among the many modes of attack that came rapidly to the fore, and one that apparently disposed of the difficulty satisfactorily, took its point of departure from one of Goethe's maxims: 'The greatest art in theoretical and practical life consists

^{1/} Harry C. Barber, Random Notes on Geometry Teaching, The Mathematics Teacher, (January 1938), Vol. 31, No. 1, p 31

^{2/} James M. Lynch, Individual Differences and Course Revision in Plane Geometry, The Mathematics Teacher, (March 1942), Vol. 35, No. 3, p 122

in changing a problem into a postulate.' Accordingly, following this procedure, it was urged that if the pupils' capacity for learning the congruence theorems, for example, is low, do not try to teach those theorems -- just postulate them, that is, treat them as axiomatic, as data self-evident immediately and requiring no proof. Is the class too dull to understand the proposition: if two straight lines are cut by a transversal, and the alternate interior angles are equal, the lines are parallel? Then postulate that proposition, too, and all annoyances and confusions about how to 'get it across' will vanish as though blown away on the wings of a gentle breeze."

Lynch^{1/} continues in the same vein: "Merely turning the problems of teaching plane geometry to the 'masses' into postulates has the rather attractive advantage of being a very pleasantly painless procedure. It has the added advantage of increasingly gaining favor and support, as a teaching device, among professors of education, curriculum experts, and textbook authors. Indeed, one of the distinctive marks of a modern progressive text is the number of postulate assumptions and assumed theorems it contains -- the greater the amount of postulation, the more 'modern' and 'progressive' it is."

^{1/} James M. Lynch, Individual Differences and Course Revision in Plane Geometry, The Mathematics Teacher, (March 1942), Vol. 35, No. 3, p. 122

Lynch prefers to attack the problem by working on the mode of presentation as an important factor in understanding; and he suggests that by changing the style of presentation rather than the content of the course, the great mass of ordinary pupils will be able to cope with the material commonly reserved for a selected few.

Further investigation into suggested omissions and additions for the course in plane geometry, brings to light the following articles: Reeve^{1/} in A Proposal for Mathematics Education in the Secondary Schools of the United States says "In geometry it will be necessary to omit: (1) About two-thirds of the traditional propositions to be proved fully. The real purpose of logical geometry can better be secured by retaining only the necessary basal propositions, introducing more original matter, and reducing the deduction aspects of the course for many pupils." "In geometry we should add: (1) A modern beginning, establishing the truth of the propositions informally. This movement is already under way."

On choosing the theorems to keep, Samuel Welkowitz^{2/} has this to say, "The point to be stressed is not ground to

^{1/} William D. Reeve, A Proposal for Mathematics Education in the Secondary Schools of the United States, The Mathematics Teacher (January 1943), Vol. 36, No. 1, pp 11-20

^{2/} Samuel Welkowitz, Tenth Year Geometry for all American Youth, The Mathematics Teacher (March 1946), Vol. 39, No. 3, pp. 99-112

be covered but ground to be cultivated. More emphasis should be placed on the nature and meaning of deductive reasoning or the process of drawing necessary conclusions from a given set of assumptions and its application in all life thinking. Less emphasis should be placed on the solving of originals and more attention should be given to the appreciation of the nature of reasoning and types of reasoning.

"In line with the above the following guiding principles are recommended in selecting the propositions to be retained. Only those propositions should be retained for deductive proof or for factual knowledge or both which fulfill at least one of the following conditions: (1) They have many varied and interesting applications in the sciences, industry, shop, navigation and the arts of war (2) They form an indispensable link in the logical chain of reasoning. In the latter case it may sometimes be more desirable to assume the truth of the proposition on the basis of an informal proof or experiment."

I would like to close this discussion of the literature on the subject by reviewing some of the remarks of Rolland R. Smith^{1/} on "How Much Formal Proof in Plane Geometry?" In this, he states, "We

^{1/} Rolland R. Smith, How Much Formal Proof in Plane Geometry? Subject of talk given at open conference on the teaching of mathematics in secondary schools at Boston University, School of Education, April 20, 1949

should have enough formal proof to fulfill our aims." In order to do this we should clarify our aims and he suggests that five or six large aims are certainly better than fifty or sixty small aims.

The changed type of College Entrance Board Examinations in mathematics has been responsible for some of the change in organization of material in plane geometry. Twenty years ago this examination would have consisted of six questions, two of them being book theorems to be proved. Since 1933, however, the examinations have been quite different. No proofs have been required for several years, but students have had to reason.

His point of view is best expressed by his statement of how to handle proof of propositions in classes of varying ability. He says, "Discuss the proof informally in all classes and reproduce it in the best classes."^{1/}

^{1/} Rolland R. Smith, How Much Formal Proof in Plane Geometry? Subject of talk given at open conference on the teaching of mathematics in secondary schools at Boston University, School of Education, on April 20, 1949

CHAPTER II

EXPERIMENTAL PROCEDURES

The two groups to be experimented with consist of two classes each of eleventh-grade students enrolled in a traditional plane geometry course. One group, during the school year 1947-48, was taught by the formal method with the formal proof of theorems; e.g. the congruence of triangles by superposition learned or committed to memory. The other group, during the school year 1948-49, meeting at the same hours of the day, having the same teacher and textbook, used the informal method where the formal proof of the same theorems was omitted and the theorems were postulated for future use in solution of original examples.

These groups were taken from a junior-senior high school of about four hundred and fifty pupils total enrollment. A check with the school office showed a great deal of information to be available from the cumulative record cards as to previous grades, chronological age, ability, etc.

In addition this school has adopted the policy of using the Boston University School and College Relations Cooperative Testing Service which tests all pupils in the eighth grade with an educational battery of tests and again in the eleventh grade with a vocational battery of tests. Results of these tests are available for both groups

as eleventh graders, but for only the 1949 group as eighth graders. This would appear to be sufficient, however; and while some further refinement in grouping might have been possible with the eighth-grade data for both groups, this cannot be had for another year.

This data will be available in the following tables and charts in Appendix B:

1. Graph of Intelligence Quotient versus Frequency of All Eleventh-grade Students and of Plane Geometry Students in 1948.
2. Graph of Intelligence Quotient versus Frequency of All Eleventh-grade Students and of Plane Geometry Students in 1949.
3. Data Used in Calculating Mean Intelligence Quotients and Sigma of Unequated Groups.
4. Data Used in Calculating Mean Intelligence Quotients and Sigma of Equated Groups - First Trial.
5. Data Used in Calculating Mean Intelligence Quotients and Sigma of Equated Groups.
6. Intelligence Quotients and Previous Marks in Mathematics of 1948 Group.
7. Intelligence Quotients and Previous Marks in Mathematics of 1949 Group.
8. Data from Boston University School and College Relations Cooperative Testing Service Vocational Battery for Pupils in 1948 Group.
9. Data from Boston University School and College Relations Cooperative Testing Service Vocational Battery for Pupils in 1949 Group.

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In setting up the experiment, it was proposed, first, to equate the groups by pairing of individuals. This was done by plotting intelligence quotients versus the average of mathematics grades for the 1948 and 1949 groups as shown in Chart 3, Appendix A. The results were not satisfactory, however, as a study of Chart 1 will show. First, in regard to intelligence quotients, note that there is only one horizontal pair line indicating the same ability level; and in twenty-four of the remaining thirty-four pairs, the 1948 group was below the 1949 group. Comparing the average of mathematics marks for grades seven, eight, nine, and ten, we find nine vertical pair lines indicating the same average grades; and in seventeen of the remaining twenty-six pairs, we find the 1948 group below the 1949 group.

Since the pairing of individuals produced only about thirty-five pairs of the forty odd pupils in each group compared, and a study of the pairs showed that the 1949 group had decided advantages, it was then decided to attempt to equate by groups.

Taking the data available for all students in each group, it was found that the mean intelligence quotient for the 1948 group was 105 with a sigma of 12.05, and for the 1949 group the intelligence quotient was 112

with a sigma of 11.98. This difference between the means was checked for significance by the formula

$$\text{Sigma}_D = \sqrt{\text{Sigma}_{M_{49}}^2 + \text{Sigma}_{M_{48}}^2}$$
 (see calculations Table 4, Appendix A). The standard error of the difference was found to be plus or minus 2.71.

Dividing the difference between the means by the standard error of the difference gave a ratio of 2.76, which from Table 34, page 213 in Garrett on Statistics^{1/} showed that the chances were 99.72 in 100 or almost a virtual certainty that the difference between the two groups was significant.

The above results necessitated the elimination of some of the lower members of the 1948 group and of the higher members in the 1949 group. The first trial of dropping the four lowest members of the 1948 group and the top two of the 1949 group gave a mean intelligence quotient of 106 for 1948 against 114 for 1949. On the second trial dropping the lowest nine for 1948 and the top four of 1949 gave a mean intelligence quotient of 108, sigma 10.2, for 1948 versus 110, sigma 10.1 for 1949. This difference between the means was then checked for significance by the same type of computation as the original difference (see Table 5, Appendix A), and the standard error of the

^{1/} Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940, p. 213

difference was found to be plus or minus 2.43. Dividing the difference between the means by the standard error of the difference gave a ratio of .783, which from Table 34^{1/} entering at .80 showed that the chances were reduced to 79 in 100 that the true difference was greater than zero. Since the lower limit is negative, there is some chance that the true difference is less than zero. Inasmuch as the authors of the test give the probable error of estimate based on 600 pupils to be four points, this difference does not seem to be too great.

This now left thirty-five members in each group of plane geometry students with which to carry on the experiment.

These students have been equated upon the basis of intelligence quotient, but in line with the suggestion in Whitney^{2/} Elements of Research their equivalence has been checked for certain other significant characteristics summarized in the following table taken from the Data Tables in Appendix B.

^{1/} Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940, p. 213

^{2/} Frederick L. Whitney, The Elements of Research, New York, Prentice-Hall, Inc., 1942, p. 225

Table I. Summary of Equivalence Factors

Factor	1948 Group Means	1949 Group Means	Appendix B Tables
Intelligence Quotient	108	110	5
Chronological Age	16 yrs 2 mos	16 yrs 0 mos	8 & 9
Average of Previous Mathematics Marks	2.43 - C+	2.47 - C+	6 & 7
Problem Solving Ability	7	7	8 & 9
Reading Comprehension	165	165	8 & 9
Spatial Relations	46	40	8 & 9

The next question that arose was what theorems should be postulated. The authorities in the field, Christofferson, Shibli, Herberg and Orleans, Birkhoff and Beatley, etc., are very much in agreement as to the main groups that should be postulated.

W.D.Reeve in the Fifth Yearbook^{1/} suggests the following be postulated: first, congruence of triangles; second, equality of alternate interior angles of parallel lines; third, theorems on similar figures; fourth, Pythagorean Theorem; fifth, measurement of angle between two chords.

^{1/} National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University), p.23

For this experiment, it was decided that the group of theorems whose proofs depend upon artificial means such as superposition and indirect proof should be postulated. The second criteria applied was whether the theorems were needed in the proof of other theorems. Table 6, Appendix A gives the complete list of theorems postulated with the 1949 group. All other theorems were handled as original exercises. One interesting feature of the textbook used was that in the group of original exercises immediately preceding a theorem, in many instances, an exercise was used which was identical with the following theorem.

Of the textbooks examined (see Table 7, Appendix A) several were found which postulated or assumed the proof of practically all theorems. Several more were found which assumed or postulated the proof of the congruence theorem; but only one, Herberg and Orleans^{1/} A New Geometry for Secondary Schools, followed the pattern outlined in the previous paragraph. The textbook used by the experimental group, Welchons & Krickenberger^{2/} Plane Geometry, presented

^{1/} Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston, D.C. Heath and Co., 1948)

^{2/} A.M. Welchons and W.R. Krickenberger, Plane Geometry, (Boston, Ginn and Co., 1943)

a difficulty that could have been eliminated if the Herberg and Orleans text could have been used. Proofs of the assumed theorems were omitted from the body of the text but were given in a section at the end of the book.

This led to some discussion among the better students as to why the work was being omitted. In order not to bias this part of the experiment, the few who wanted to go into proofs such as superposition and having time to do so were told why these were being postulated and advised to go ahead on their own study of formal proofs and the instructor would be available for discussions outside of class.

There seem to be three main groups of textbooks with regard to this idea of postulating or assuming the truth of a theorem without formal proof: First, the group of which Schnell and Crawford^{1/} Clear Thinking may be said to be typical where many assumed theorems are accepted and few formal proofs appear in the book; second, the group of which Herberg and Orleans^{2/} A New Geometry for Secondary Schools is typical taking a middle ground

^{1/} Leroy H. Schnell and Mildred Crawford, Clear Thinking, An Approach through Plane Geometry, (New York, Harper & Brothers, 1943)

^{2/} Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston, D.C. Heath and Co., 1948)

between the formal and informal and while theorems are postulated, complete proofs are provided in the back of the book; third, the group of which Seymour and Smith^{1/} Plane Geometry is typical in which the congruence of triangles theorems and a few others are postulated. A list of geometry books falling in the various groups will be found in Table 7, Appendix A.

While the main factor on which the groups were equated was that of ability rating as determined by the California Test of Mental Maturity^{2/}, Advanced form S, there were other factors which needed to be taken care of so that the one variable experiment could be carried on. Among these, the timetable of work for 1948 versus 1949 classes (see Table 8, Appendix A) was watched closely. The Teacher's Plan Book for 1948 was the source used for setting up this table. The work of the 1949 group was held as closely to that of the 1948 group as possible so that no undue advantage would be given either group. This was accomplished by allotting the same number of days to the 1949 group for each unit of work as was required by the 1948 group. The timetable of work for the 1948

^{1/} F. Eugene Seymour and Paul J. Smith, Plane Geometry, Boston, Macmillan Co., 1941)

^{2/} Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, California Short-Form Test of Mental Maturity, Advanced S-Form, (California Test Bureau, Los Angeles, 1939)

group versus the 1949 group gives the following information: the units in the order taught; number of days required by the 1948 group; number of days actually used by the 1949 group; a column showing the difference in time actually used by the two groups, a plus indicates less time required and a minus indicates more time required for the 1949 group as compared to the 1948 group.

It is interesting to note that due to the informal method used, a gain of about fifteen days' time was made during the first half year. Ten of these days were allotted to a unit on "space geometry" which had previously been left out of the course. The remaining five days were kept for possible use during the second half of the year's work. This time came in very handy as the group became so interested in the subject of loci constructions that an additional week was allotted to this unit of work.

The next problem to confront the writer was to determine whether the tests given to the 1948 group at the half year and at the end of the year could be used satisfactorily with the 1949 group for the desired result.

Since the work with the 1948 group had already been closed no change could be made in the examinations

given them. If an ideal experimental set up could be realized; that is, two large groups carried on at the same time, this problem would not have arisen.

The 1948 group was examined in January of 1948 after approximately one-half year of study of plane geometry by the Orleans^{1/} Plane Geometry Achievement Test 1: Form A for the first half year. This consisted of five parts: geometrical reasons, computation, completing proofs, diagrams, and original proofs. The author states: "The validity of the test is shown by the fact that the correlation between test scores and teachers' school marks for ten different teachers vary from .66 to .88, the median being .81. The high validity is due in a large part to the fact that each test covers the work of only one semester and, therefore, covers it thoroughly. It is also due to the analytical organization of the test and the fact that its content and organization are aimed at the work taught in the usual geometry class." The authors also go into a discussion of how the various forms of the test were standardized, and state "The reliability of the test is shown by the correlations between Form A and Form B for ten teachers vary from .77

^{1/} Joseph B. Orleans and Jacob S. Orleans, Orleans Plane Geometry Achievement Test, (World Book Company, New York, 1929)

to .87 for Test 1, with a median correlation of .85; and from .58 to .80 for Test 2, with a median correlation of .71." Percentile norms are available based on 3500 cases.

The final examination for the 1948 group given in June of 1948 was determined by the practice of the school where the experiment was carried on. Cooperative Tests of the American Council on Education are required in all subjects that tests are available for. The plane geometry classes were examined by the Cooperative Plane Geometry Test^{1/}, Revised Series Form R, by Long, Siceloff, and Spaney. This test consists of three parts: Part one, thirty true-false statements; Part two, twenty multiple choice problems with five answers to select from; Part three, fifteen more difficult multiple choice problems with five answers to select from. Norms were available based on 9000 students from ninety schools.

The following comments covering the tests chosen for use in the experiment were taken from The Third Mental Measurements Yearbook^{2/}: In a review of the Orleans Plane Geometry Achievement Test, Fawcett^{3/} states

^{1/} John A. Long, L.P. Siceloff, and Emma Spaney, Cooperative Plane Geometry Test, Revised Series Form R, (Cooperative Test Service, New York, 1941)

^{2/} Oscar K. Buros, The Third Mental Measurements Yearbook, Rutgers University Press, New Brunswick, 1949)

^{3/} Harold P. Fawcett, Review of Orleans Plane Geometry Achievement Test and Cooperative Test, The Third Mental Measurements Yearbook, (Rutgers Univ. Press, New Brunswick, 1949), pp 357-362

"The test measures the factual content of geometry but ignores the larger values associated with the nature of a proof." His comment on the Cooperative Plane Geometry Test Forms R, S, and T was that they cover the facts of geometry and measure them well but, like the Orleans Test, there is not enough on the nature of a proof.

Oakley^{1/} in his review of the Cooperative Plane Geometry Tests says "the test adequately covers material usually given in a plane geometry course and is well designed with respect to range and difficulty.

Because of the fact that different tests were given at the half year and the end of the year, it was necessary in order to compare the two scores to determine the gain in achievement to convert the scores on both tests to T-scores. T-scores are expressed in the same units and with respect to the same zero point and are equal throughout the scale. Hence T-scores from different tests are directly comparable and may be averaged or combined by simple addition.

1/ C.O.Oakley, Review of Cooperative Plane Geometry Tests, Forms R, S, T., The Third Mental Measurements Yearbook, (Rutgers University Press, New Brunswick, 1949) 357-362

The following Tables in Appendix A contain the data and graphs used in comparing the achievement of the two groups:

- Table 9. Semi-Final and Final Test Scores for 1948 and 1949 Groups.
- Table 10. Conversion of 1948 Semi-Final Scores to Sigma Scale and T-Scores.
- Table 11. Conversion of 1949 Semi-Final Scores to Sigma Scale and T-Scores.
- Table 12. Conversion of 1948 Final Scores to Sigma Scale and T-Scores.
- Table 13. Conversion of 1949 Final Scores to Sigma Scale and T-Scores.
- Table 14. Significance of the Differences between the Means on Semi-Final and Final Test Scores.
- Table 15. Comparison of Scores and Sigma_D for Upper Halves of Groups on Semi-Final Tests.
- Table 16. Comparison of Scores and Sigma_D for Lower Halves of Groups on Semi-Final Tests.
- Table 17. Comparison of Scores and Sigma_D for Upper Halves of Groups on Final Tests.
- Table 18. Comparison of Scores and Sigma_D for Lower Halves of Groups on Final Tests.
- Table 19. Graphs of Achievement on Semi-Final Tests for 1948 and 1949 Groups.
- Table 20. Graphs of Achievement on Final Tests for 1948 and 1949 Groups.

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CHAPTER III

GENERAL SUMMARY

In summing up, let us first consider the two groups with which we experimented. Tables 1 and 2 of Appendix B were developed with the idea of showing the relationship of the groups used in the experiment to the larger groups of which they were members. The curve for the 1948 eleventh grade shows a median Intelligence Quotient of 101 for the seventy-six members of the grade with a median Intelligence Quotient of 107 for the plane geometry group. Of the frequency groups above the median, the plane geometry group contained a majority of the members of the entire class and below the median a much smaller membership of the entire class was found in each frequency group of the plane geometry class. This was also true for the 1949 geometry groups as compared to the 1949 eleventh grade.

These groups then can be said to show a pattern which could reasonably be expected to appear for all plane geometry groups when compared to the larger grades of which they are a part and contain most of the members of the higher Intelligence Quotient frequency groups and fewer members of the lower Intelligence Quotient frequency groups.

The attempt to equate the groups by pairing of individuals has been discussed at length in the previous

chapter; and while the same number of pairs (thirty-five) were plotted in Table 3 of Appendix A as the number of individuals kept when finally equated by groups, a glance at the summaries given on Table 3 shows a decided advantage for the 1949 group when paired with the 1948 group.

In attempting to equate by groups (see Table 4, Appendix A), it was first found that in keeping all members of each geometry group a difference of 7.48 between the means and in favor of the 1949 group was found. This difference divided by the standard error of the difference gave a ratio of 2.71 and is interpreted as a significant difference between the means. After several attempts, results of which are given in Tables 3, 4, and 5 of Appendix B, of dropping higher members of one group and lower members of the other group, a group of thirty-five members in each experimental group was chosen. The difference between the means divided by the Sigma_D (see Table 5, Appendix A) gave a ratio of .78. Garrett^{1/} says "It is customary to take a difference divided by Sigma_D of 3.0 as indicative of a significant difference (virtual certainty) since there is only about one chance

^{1/} Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), p. 213

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in 1000 that a difference of +3 Sigma will arise when the true difference is zero." Following this line of reasoning our ratio of .78 is about one-quarter of what it should be, namely, 3.0, to insure a significant difference and from Garrett^{1/} Table 34 entering at .80 we find the chances 79 in 100 that the true difference is greater than zero.

While the main factor on which the groups were equated was Intelligence Quotient, the following sub-factors were checked: chronological age; previous mathematics marks; three scores from the Boston University Cooperative Test Service Vocational Battery (Problem Solving, Reading Comprehension, and Spatial Relations). These have been summarized in Chapter II in a Table of Equivalence Factors. In this table the following differences of the means are noted: chronological age, two months in favor of 1948 groups; average of previous mathematics marks, no difference in letter grades assigned; problem solving ability, no difference in mean scores; reading comprehension, no difference in mean scores; spatial relations, six points in favor of 1948 group. On all of these except the last, the differences are so small as to be considered negligible. The final sub-factor, spatial relations, where a greater difference

^{1/} Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), Table 34, p 213

existed, might well be the deciding factor which influenced the results in favor of the 1948 group.

The data used in comparing the achievements of the two groups was obtained from the scores on the semi-final and final tests given to both groups and presented in Table 9 of Appendix A. In order to facilitate the comparison, all scores on the semi-final tests and final tests were converted to Sigma scale and T-scores and are contained in Tables 10 through 13.

Tables 19 and 20 show respectively graphs of 1948 and 1949 group achievement on semi-final tests and 1948 and 1949 group achievement on final tests. A study of these graphs brought up the question as to whether either of the methods used showed any significant difference in the halves of the group with high ability and also in the halves with low ability and led to the development of Tables 15 through 18.

The question is answered by a study of these tables which show that while slight differences exist in favor of the upper and lower halves of the 1948 group over the corresponding halves of the 1949 group, they are not significant.

Table 14 of Appendix A contains the calculation of the significance of the difference between the means on the semi-final and final tests. While some difference

existed between the means in favor of the 1948 group on both tests, the difference divided by the sigma of the difference giving ratios of 1.8 on the semi-final tests and 1.1 on the final tests indicates that these differences are not significant. The ratio of D to Sigma_D for the final test of 1.1 was about one-third of what it should be; namely, 3.0 to indicate a significant difference, and that of the semi-final of 1.8 about three-fifths of what it should be. These ratios using Garrett^{1/} Table 34 show that the chances are 86 in 100 that the difference is significant on the final test and 96 in 100 that the difference is significant on the semi-final test.

The table on the following page summarizes the calculations of Sigma_D for Tables 14 through 18.

The method used in this table for summarizing the results of the experiment was discovered first in a research paper by Hunziker and Douglass^{2/} entitled, The Relative Effectiveness of a Large Unit Plan of Supervised Study and the Daily Recitation Method in the Teaching of Algebra and Geometry. It was also used extensively in a

^{1/} Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940), p. 213

^{2/} C.W. Hunziker and H.R. Douglass, The Relative Effectiveness of a Large Unit Plan of Supervised Study and the Daily Recitation Method in the Teaching of Algebra and Geometry, The Mathematics Teacher (March 1937), Vol. 30, No. 3, pp. 122-124

Table II.

Summary of Tables 10 - 18

Test	1948 Group Mean	1948 Group Sigma	1949 Group Mean	1949 Group Sigma	Difference of Means	$\frac{D}{\text{Sigma}_D}$	Chances in 100
Semi-final	45	13.1	39	14.7	6	1.8	96
Final	58	11.2	55	11.3	3	1.1	86
Upper Half Semi-final	51	9.7	49	10.5	2	.58	73
Lower Half Semi-final	37	12.3	30	12.3	7	1.6	94
Upper Half Final	63	10.5	62	7.3	1	.33	64
Lower Half Final	52	9.8	49	9.2	3	.93	83

The above Data: Lines one and two from Appendix A, Tables 10 - 14; lines three through six from Tables 15 - 18.

master's thesis by Nielsen^{1/} entitled, Permanent Outcomes from Teaching Plane Geometry by Two Different Methods.

One of the original premises of this paper was to compare the achievement of the two groups by the gain in achievement from semi-final to final tests. This I attempted to do graphically, but the results were not satisfactory for the purpose of drawing conclusions. Upon the suggestion of the First Reader, the table on the following page was worked up wherein the means and sigmas of the combined groups, 1948 and 1949, were presented for both tests.

This now gave a larger group with which to compare the experimental groups. Since there were the same number of pupils in each group, the mean of the combined group was easy to obtain. The sigma of the combined group was obtained by the following formula from Garrett^{2/}:

$$\text{Sigma}_{\text{Combined}} = \sqrt{\frac{N_1 (\text{Sigma}^2_1 + d^2_1) + N_2 (\text{Sigma}^2_2 + d^2_2)}{N}}$$

Dividing the difference between the group means and the combined mean by the sigma of the combined group,

^{1/} M.R.Nielsen, Permanent Outcomes from Teaching Plane Geometry by Two Different Methods, Master's Thesis, Iowa State College, 1938.

^{2/} Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), p. 192

Table III. Comparison of Gains in Achievement
From Semi-Final to Final Tests

	Semi-Final	Sigma Scale	Final	Sigma Scale	Gain
Mean ₄₈	45		58		
Sigma ₄₈	13.1	+.21	11.2	+.09	-.12
Mean ₄₉	39		55		
Sigma ₄₉	14.7	-.21	11.3	-.18	+.03
Mean _C	42		57		
Sigma _C	14.3		11.4		

Calculation needed for combined group in above table:

$$\text{Semi-Final Mean} = \frac{45 + 39}{2} = 42$$

$$\text{Final Mean} = \frac{58 + 55}{2} = 56.5 = 57$$

$$\text{Semi-Final Sigma}_{\text{Combined}} = \sqrt{\frac{35 (13.1^2 + 3^2) + 35 (14.7^2 + (-3)^2)}{70}}$$

$$\text{Sigma}_{\text{Combined}} (\text{Semi-Final}) = 14.3$$

$$\text{Final Sigma}_{\text{Combined}} = \sqrt{\frac{35 (11.2^2 + 1^2) + 35 (11.3^2 + (-2)^2)}{70}}$$

$$\text{Sigma}_{\text{Combined}} (\text{Final}) = 11.4$$

we obtained ratios which were used for comparison of the groups.

On the semi-final test, the 1948 group was .21 sigma above the mean and the 1949 group .21 sigma below the mean. On the final test, the 1948 group was only .13 sigma above the mean and the 1949 group was .13 sigma below the mean. The last column is interpreted to indicate that the 1948 group showed a slight loss while the 1949 group showed a slight gain.

An interesting sidelight of the experiment was the development of the Classification Table of Types of Plane Geometry Textbooks (Table 7, Appendix A). While there are only a few books included in each of the first three groups, about half of these are now published as revised or second editions. Note particularly the copyright dates of the first three groups which, with the exception of the first book in group three, first appeared about 1938-1940. Group four with by far the greatest number of books represented in it covers a period of time from 1915-1948. A closer examination of these textbooks and any others which might be available might well be the subject of another service paper.

CHAPTER IV

CONCLUSION AND RECOMMENDATIONS

Results of this experiment show no significant difference between the formal and informal methods relative to achievement in factual information and skills in reasoning about geometric situations.

The informal method required less time than the formal method to cover the units of work in the plane geometry course. The time saved was used for units in space geometry and loci constructions, topics which have invariably been left out of the usual course in plane geometry for lack of time.

Comparing the groups as a whole, there was a slight difference in favor of the group using the formal method; statistically this was not found to be significant.

Comparing the upper and lower halves of the groups graphically (Tables 19 and 20), there was a difference in favor of the informal method for the upper halves of the groups in both semi-final and final tests. This contrasted with the lower halves of the groups where the difference was in favor of the formal method on both semi-final and final tests. All differences between whole and half groups were found to be in favor of the 1948 group, but

were not significant. We may well conclude that despite the material left out (proof of propositions) there was no loss in achievement by the informal group whose achievement was very close to that of the formal group.

This study does not show that either the formal method or the informal method has any measurable difference in effect upon pupil achievement, and while no statement can be made on the statistical evidence as to which is the better method of teaching plane geometry; nevertheless, the writer feels that the saving in time alone gives the informal method an advantage over the informal method.

No decision can be made from the statistical results of this experiment as to the choice of type of textbook to be used, but the writer believes that an informal-type textbook would increase the amount of time available to be used for additional units.

The results of this experiment as expressed in the conclusions are not what the writer expected. While holding a slight prejudice in favor of the informal method, every effort was made to keep this from influencing the experiment and certainly the results bear this out.

Let us next consider whether the results of this experiment are reasonable or not. A comparison of the

graphs of Tables 19 and 20 in Appendix A show very clearly that the same pattern of achievement existed on both the semi-final and final tests. The difference between the two curves on each table are small and might well be considered to indicate no difference in achievement between the groups regardless of the method used.

There are many factors which influence an experiment of this type, not all of which can be brought into the equating process. I have in mind an incident which happened with the 1949 group who were taking their final tests during a period of exceptionally hot, humid weather. This might well be considered to have influenced their achievement on this test.

It is recommended that this experimental paper be used as a pilot for a future experiment with the following factors being carefully checked:

1. Larger groups are needed for better equating.
2. Both methods should be used in the same school year with rotation of groups.
3. A pre-test should be given to determine what geometrical factors and skills the groups possessed at the beginning of the experiment.

4. Use the same test or other forms of the same test for the three test periods; pre-test, semi-final, and final.

5. Use an informal-type book (see Table 7, Appendix A) with groups using the informal method.

APPENDIX A

Table 1. A Comparison of Axioms and Postulates; Theorems, Corollaries, and Constructions of six often-used Textbooks. ^{1/}

Name of Author	Axioms and Postulates of Plane Geometry	Theorems, Corollaries and Constructions of Plane Geometry	Axioms and Postulates of Solid Geometry	Theorems and Corollaries of Solid Geometry
Durrell and Arnold	38	196	11	153
Nyberg	52	198	15	127
Otis Clark	38	165	9	114
Seymour	48	218		
Smith, Foberg, and Reeve	33	162	16	59
Wells, Hart	29	231		
Average	40	195	12	131
This Study	32	30	4	12

Note: There may be errors in the above count because of cases where it was not clear whether or not the author intended statement for a definition, a postulate, or a theorem. Some authors also used principles, properties, and problems; and again it was not clear what these were intended for.

^{1/} H.C.Christofferson, Geometry Professionalized for Teachers, George Banta Publishing Company, Menasha, Wisconsin, 1933, Table LV, p. 43

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Table 2. Suggested Places in Plane Geometry in which we can Shorten, Eliminate, or Postulate to Allow Time to Introduce Space Concepts. 1/

-
1. We may omit, to be included later, the theorem, "The exterior angle of a triangle is greater than either opposite interior angle."
 2. It is suggested that we postulate the theorem "One and only one line can be drawn from (at) a given point perpendicular to a given line."
 3. If the hypotenuse - angle congruency proposition is proved by superposition, it might be postulated.
 4. It might be possible to postulate the theorem "If two angles have their sides respectively perpendicular, they are either equal or supplementary."
 5. The theorem "The sum of the angles of a polygon of n sides is $(n-2)$ straight angles" may be postulated or proved informally.
 6. In certain texts it will be found possible to do some reversing and some combining of the theorems on the properties of a parallelogram.
 7. In connection with inequalities in a triangle or a circle, certain theorems and corollaries might be postulated.
 8. Group the theorems concerning interdependence of central angles, their arcs and chords; interdependence of diameters and perpendicular bisectors or chords; interdependence of tangents and radii to the points of contact.
 9. The concurrency propositions, having been intuitively established in the introduction, may be postulated.
 10. Eliminate harmonic division of a line segment as a separate treatment.
 11. Substitute the algebraic for the geometric proof of the Pythagorean Theorem.

Table 2. (continued)

12. Eliminate the generalized Pythagorean Theorems referring to acute and obtuse triangles.
13. Eliminate the proof of Hero's formula or make it a matter for class discussion only.
14. Postulate the theorem "If two polygons are similar, they can be divided into triangles which are similar and similarly placed" and its converse.
15. Postulate the continuity of the Pythagorean Theorem as applied to similar polygons constructed upon the three sides of a right triangle.

1/ From Report of Second Annual Purdue Mathematics Workshop Committee on first six weeks in Plane Geometry, June 16 - 28, 1947

Table 3. Chart Showing Pairs of 1948 and 1949 Plane Geometry Students, Plotting Intelligence Quotients versus Average of Mathematics Marks

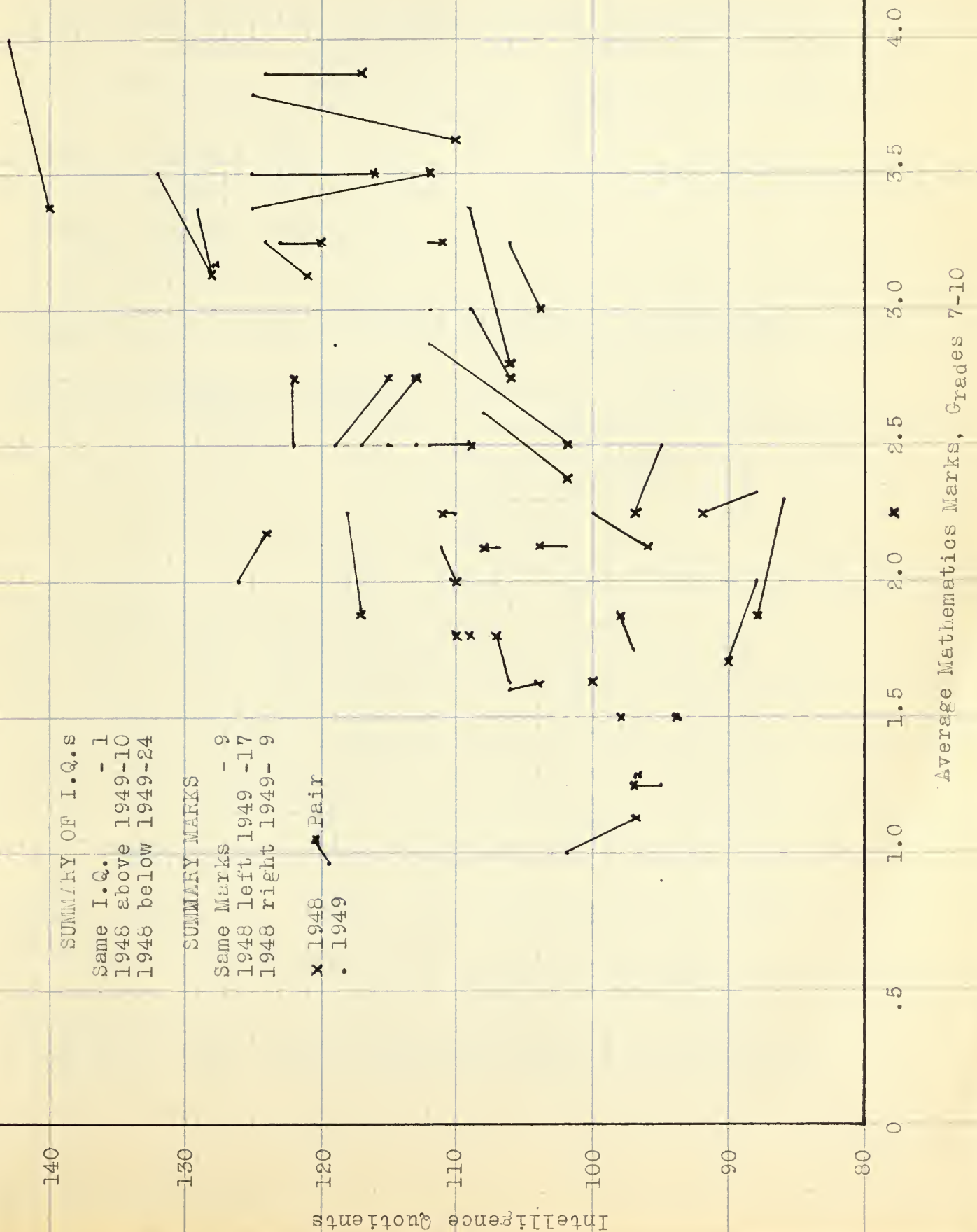




Table 4. Significance of the Difference Between the Means - Unequated Groups

1948 - 44 students	Mean 104.56 = 105	Sigma = 12.05 = 12.1
1949 - 39 students	Mean 112.05 = 112	Sigma = 11.98 = 12.0

$$\text{Sigma}_{M_{48}} = \frac{12.1}{\sqrt{44}} = \frac{12.1}{6.63} = 1.91$$

$$\text{Sigma}_{M_{49}} = \frac{12.0}{\sqrt{39}} = \frac{12.0}{6.25} = 1.92$$

Standard error of the difference between two uncorrelated means.

$$\begin{aligned} \text{Sigma}_D \text{ or } \text{Sigma}_{M_{49}} - \text{Sigma}_{M_{48}} &= \sqrt{\text{Sigma}_{M_{49}}^2 + \text{Sigma}_{M_{48}}^2} \\ &= \sqrt{1.91^2 + 1.92^2} \\ &= \sqrt{7.3345} \\ \text{Sigma}_D &= 2.71 \end{aligned}$$

Significance of the difference between the means.

$M_{49} - M_{48} = 7.48$. Chances are 68 in 100 that the difference of 7.48 does not differ from the true difference by more than ± 2.71 and chances are 99 in 100 that the difference of 7.48 does not differ from the true difference by more than $\pm 3 \times 2.71 = \pm 8.13$ and lies between $-.65$ and $+15.61$.

$\frac{D}{\text{Sigma}_D} = \frac{7.48}{2.71} = 2.76$. Using 2.76, from Garrett^{1/}, chances are 99.72 (100) or almost virtual certainty that the difference is greater than zero between the two groups.

^{1/} Henry E. Garrett, *Statistics in Psychology and Education*, Longmans, Green and Co., New York, 1940, Table 34, p. 213

Table 5. Significance of the Difference Between
the Means - Equated Groups

1948 - 35 students	Mean 107.8 = 108	Sigma = 10.24 = 10.2
1949 - 35 students	Mean 109.7 = 110	Sigma = 10.09 = 10.1

$$\text{Sigma}_{M_{48}} = \frac{10.2}{\sqrt{35}} = \frac{10.2}{5.91} = 1.73$$

$$\text{Sigma}_{M_{49}} = \frac{10.0}{\sqrt{35}} = \frac{10.1}{5.92} = 1.70$$

Standard error of the difference between two uncorrelated means.

$$\begin{aligned} \text{Sigma}_D \text{ or } \text{Sigma}_{M_{49}} - \text{Sigma}_{M_{48}} &= \sqrt{\text{Sigma}_{M_{49}}^2 + \text{Sigma}_{M_{48}}^2} \\ &= \sqrt{1.73^2 + 1.70^2} \\ &= \sqrt{5.8829} \\ \text{Sigma}_D &= 2.43 \end{aligned}$$

Significance of the difference between the means.

$M_{49} - M_{48} = 1.9$. Chances are 68 in 100 that the difference of 1.9 does not differ from the true difference by more than ± 2.43 and chances are 99 in 100 that the difference of 1.9 does not differ from the true difference by more than $\pm 3 \times 2.43 = \pm 7.29$ and lies between -5.39 and +9.19.

$\frac{D}{\text{Sigma}_D} = \frac{1.9}{2.42} = .78$. Using .80, from Garrett^{1/}, chances are 79 in 100 that the true difference is greater than zero. Since the lower limit is negative (-5.38), there is some chance that the true difference is less than zero.

^{1/} Henry E. Garrett, Statistics in Psychology and Education, Longmans, Green and Co., New York, 1940, Table 34, p. 213

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DEPARTMENT OF CHEMISTRY
JANUARY 1958

TO THE HONORABLE CHAIRMAN OF THE BOARD OF TRUSTEES
OF THE UNIVERSITY OF CHICAGO
FROM THE DEPARTMENT OF CHEMISTRY
RE: [illegible]

[The following text is extremely faint and largely illegible. It appears to be a formal report or letter, possibly detailing a chemical discovery or research findings. It contains several paragraphs of text, some of which may be headings or subheadings, but the specific content cannot be accurately transcribed.]

Table 6. Theorems Postulated from Welchons & Krickenberg
Plane Geometry

-
-
1. If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, the triangles are congruent.
 2. If two triangles have two angles and the included side of one equal respectively to two angles and the included side of the other, the triangles are congruent.
 3. If two triangles have three sides of one equal respectively to three sides of the other, the triangles are congruent.
 4. An exterior angle of a triangle is greater than either non-adjacent interior angle.
 5. If two lines form equal alternate interior angles with a transversal, the lines are parallel.
 6. If two parallels are cut by a transversal, the alternate interior angles are equal.
 7. In a circle or in equal circles equal central angles have equal arcs.
 8. In a circle or in equal circles equal arcs have equal central angles.
 9. If a line is tangent to a circle, it is perpendicular to the radius drawn to the part of contact.
 10. If a line is perpendicular to a radius at the point on a circle, the line is tangent to the circle.
 11. If a line is parallel to one side of a triangle and intersects the other two sides, it divides these sides proportionally.
 12. If a line divides two sides of a triangle proportionally, it is parallel to the third side.
 13. If two triangles have two angles of one equal respectively to two angles of the other, the triangles are similar.

Table 6. (continued)

14. If two triangles have their sides respectively proportional, they are similar.

15. A circle can be circumscribed about any regular polygon.

16. If two triangles have two sides of one equal respectively to two sides of the other and the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.

17. If two triangles have two sides of one equal respectively to two sides of the other and the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second, the angle opposite the third side of the first is greater than the angle opposite the third side of the second.

18. In a circle or equal circles the greater of two central angles has the greater arc.

19. In a circle or equal circles the greater of two unequal arcs has the greater central angle.

20. In a circle or in equal circles the greater of two unequal chords has the greater arc.

21. In a circle or in equal circles the greater of two unequal arcs has the greater chord.

Table 7. Classification of Plane Geometry Textbooks

Title	Author	Publisher	Copyright
<u>Group I Many assumed theorems; few formal proofs.</u>			
<u>Plane Geometry</u>	Major	Scribner	1938
<u>Today's Geometry</u>	Reichgott & Spiller	Prentice-Hall	1938
<u>Basic Geometry</u>	Birkhoff & Beatley	Scott, Foresman	1940-41
<u>Clear Thinking</u>	Schnell & Crawford	Harpers	1938-43
<u>Group II Postulation of group of about twenty theorems.</u>			
<u>Plane Geometry and Its Reasoning</u>	Barber & Hendrix	Harcourt, Brace	1937
<u>A New Geometry for Secondary Schools</u>	Herberg & Orleans	D.C. Heath	1940-48
<u>Group III Postulation of Congruent Triangles</u>			
<u>Senior Math, Book II of Unified Math Series</u>	Breslich	Univ. of Chicago Press	1910, 16, 23, 27
<u>Plane Geometry</u>	Seymour & Smith	Macmillan	1941
<u>Modern School Geometry</u>	Schorling, Clark, Smith	World Book Co.	1938, 43, 48

1871

1872

1873

1874

1875

1876

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1878

1879

1880

1881

1882

1883

1884

1885

1886

1887

1888

1889

1890

Table 7 (continued)

Title	Author	Publisher	Copyright
<u>Group IV Formal proofs; no postulation of theorems.</u>			
<u>New Plane Geometry</u>	Robbins	American	1915
<u>Modern Plane Geometry</u>	Stone-Mallory	Sanborn	1929
<u>New Plane Geometry</u>	Durrell & Arnold	Merrill	1916, 17 24, 30
<u>Plane Geometry</u>	Seymour	American	1925, 31
<u>Progressive Plane Geometry</u>	Wells & Hart	D.C. Heath	1935
<u>Plane Geometry and Its Uses</u>	Mirick, Newell & Harper	Row-Peterson	1935
<u>Plane Geometry and Related Subjects</u>	Breslich	Laidlow	1910, 16, 23, 27, 35
<u>Plane Geometry</u>	Schulze-Sevenoak-Stone	Macmillan	1936
<u>Plane Geometry</u>	Morgan-Foberg-Breckenridge	Houghton Mifflin	1937
<u>Plane Geometry of Purposeful Math Series</u>	Breslich	Laidlow	1938
<u>New Plane Geometry</u>	Stone-Mallory	Sanborn	1938
<u>Plane Geometry</u>	Welchons & Krickenberger	Ginn	1933, 38 40, 43

Table 7 (continued)

Title	Author	Publisher	Copyright
<u>Fundamentals of Plane Geometry</u>	Nyberg	American	1944
<u>Plane Geometry</u>	Avery	Allyn & Bacon	1947
<u>New Plane Geometry</u>	Smith-Marino	Merrill	1948

Table 8. Timetable of Work - 1948 Group vs. 1949 Group

Unit	Days taken by 1948 Group	Days taken by 1949 Group	Difference
1. Introduction	15	11	+4
2. Triangles	21	19	+2
3. Parallels and Perpendiculars	27	19	+8
4. Constructions	5	6	-1
5. Polygons	<u>21</u>	<u>19</u>	<u>+2</u>
Half-year Totals	89	74	+15
6. Area of Polygons	19	20	-1
7. Circles, Angles, and Arcs	13	12	+1
8. Measurement of Angles and Arcs	12	12	0
9. Loci	13	20	-7
10. Proportion	11	9	+2
11. Similar Polygons	13	13	0
12. Regular Polygons and the Circle	<u>5</u>	<u>5</u>	<u>0</u>
Totals	175	165	-5
x Space Geometry	0	10	-10

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NAME		ADDRESS	
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
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90	90	90	90
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92	92	92	92
93	93	93	93
94	94	94	94
95	95	95	95
96	96	96	96
97	97	97	97
98	98	98	98
99	99	99	99
100	100	100	100

Table 9. Semi-Final and Final Test Scores for 1948 and 1949 Groups

<u>1948 Group</u>			<u>1949 Group</u>		
<u>IQ</u>	<u>Semi-final</u>	<u>Final</u>	<u>IQ</u>	<u>Semi-final</u>	<u>Final</u>
140	57	68	125	59	67
128	56	71	125	65	72
128	62	68	125	62	68
124	34	54	124	67	73
122	34	67	124	55	50
121	53	66	123	37	58
120	52	65	122	42	58
117	43	64	119	26	61
117	60	70	118	36	56
116	66	71	117	36	55
115	44	50	115	52	61
113	54	49	113	58	64
112	61	79	112	52	63
111	50	68	112	45	71
111	59	70	112	44	67
110	62	68	112	44	51
110	43	50	111	33	53
110	42	41	110	42	61
109	28	50	109	50	56
109	35	26	109	50	63
108	25	57	108	33	49
107	43	49	107	39	55
106	49	55	106	33	53
106	58	67	106	17	29
104	58	64	106	15	35
104	33	60	102	31	53
104	24	51	102	28	54
102	55	62	100	38	35
102	26	43	98	22	53
100	31	49	98	14	49

Table 9. (continued)

<u>1948 Group</u>			<u>1949 Group</u>		
<u>IQ</u>	<u>Semi-final</u>	<u>Final</u>	<u>IQ</u>	<u>Semi-final</u>	<u>Final</u>
98	34	41	97	42	55
98	34	53	97	12	43
98	24	48	95	13	48
97	24	48	95	38	57
97	53	63	86	39	23
<u>Means</u>					
108	45	58	110	39	55
<u>Sigmas</u>					
102	13.1	11.2	10.1	14.7	11.3

1900			1901		
Jan	Feb	Mar	Jan	Feb	Mar
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144
145	146	147	148	149	150
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223	224	225	226	227	228
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265	266	267	268	269	270
271	272	273	274	275	276
277	278	279	280	281	282
283	284	285	286	287	288
289	290	291	292	293	294
295	296	297	298	299	300
301	302	303	304	305	306
307	308	309	310	311	312
313	314	315	316	317	318
319	320	321	322	323	324
325	326	327	328	329	330
331	332	333	334	335	336
337	338	339	340	341	342
343	344	345	346	347	348
349	350	351	352	353	354
355	356	357	358	359	360
361	362	363	364	365	366
367	368	369	370	371	372
373	374	375	376	377	378
379	380	381	382	383	384
385	386	387	388	389	390
391	392	393	394	395	396
397	398	399	400	401	402
403	404	405	406	407	408
409	410	411	412	413	414
415	416	417	418	419	420
421	422	423	424	425	426
427	428	429	430	431	432
433	434	435	436	437	438
439	440	441	442	443	444
445	446	447	448	449	450
451	452	453	454	455	456
457	458	459	460	461	462
463	464	465	466	467	468
469	470	471	472	473	474
475	476	477	478	479	480
481	482	483	484	485	486
487	488	489	490	491	492
493	494	495	496	497	498
499	500	501	502	503	504
505	506	507	508	509	510
511	512	513	514	515	516
517	518	519	520	521	522
523	524	525	526	527	528
529	530	531	532	533	534
535	536	537	538	539	540
541	542	543	544	545	546
547	548	549	550	551	552
553	554	555	556	557	558
559	560	561	562	563	564
565	566	567	568	569	570
571	572	573	574	575	576
577	578	579	580	581	582
583	584	585	586	587	588
589	590	591	592	593	594
595	596	597	598	599	600
601	602	603	604	605	606
607	608	609	610	611	612
613	614	615	616	617	618
619	620	621	622	623	624
625	626	627	628	629	630
631	632	633	634	635	636
637	638	639	640	641	642
643	644	645	646	647	648
649	650	651	652	653	654
655	656	657	658	659	660
661	662	663	664	665	666
667	668	669	670	671	672
673	674	675	676	677	678
679	680	681	682	683	684
685	686	687	688	689	690
691	692	693	694	695	696
697	698	699	700	701	702
703	704	705	706	707	708
709	710	711	712	713	714
715	716	717	718	719	720
721	722	723	724	725	726
727	728	729	730	731	732
733	734	735	736	737	738
739	740	741	742	743	744
745	746	747	748	749	750
751	752	753	754	755	756
757	758	759	760	761	762
763	764	765	766	767	768
769	770	771	772	773	774
775	776	777	778	779	780
781	782	783	784	785	786
787	788	789	790	791	792
793	794	795	796	797	798
799	800	801	802	803	804
805	806	807	808	809	810
811	812	813	814	815	816
817	818	819	820	821	822
823	824	825	826	827	828
829	830	831	832	833	834
835	836	837	838	839	840
841	842	843	844	845	846
847	848	849	850	851	852
853	854	855	856	857	858
859	860	861	862	863	864
865	866	867	868	869	870
871	872	873	874	875	876
877	878	879	880	881	882
883	884	885	886	887	888
889	890	891	892	893	894
895	896	897	898	899	900
901	902	903	904	905	906
907	908	909	910	911	912
913	914	915	916	917	918
919	920	921	922	923	924
925	926	927	928	929	930
931	932	933	934	935	936
937	938	939	940	941	942
943	944	945	946	947	948
949	950	951	952	953	954
955	956	957	958	959	960
961	962	963	964	965	966
967	968	969	970	971	972
973	974	975	976	977	978
979	980	981	982	983	984
985	986	987	988	989	990
991	992	993	994	995	996
997	998	999	1000	1001	1002
1003	1004	1005	1006	1007	1008
1009	1010	1011	1012	1013	1014
1015	1016	1017	1018	1019	1020
1021	1022	1023	1024	1025	1026
1027	1028	1029	1030	1031	1032
1033	1034	1035	1036	1037	1038
1039	1040	1041	1042	1043	1044
1045	1046	1047	1048	1049	1050
1051	1052	1053	1054	1055	1056
1057	1058	1059	1060	1061	1062
1063	1064	1065	1066	1067	1068
1069	1070	1071	1072	1073	1074
1075	1076	1077	1078	1079	1080
1081	1082	1083	1084	1085	1086
1087	1088	1089	1090	1091	1092
1093	1094	1095	1096	1097	1098
1099	1100	1101	1102	1103	1104
1105	1106	1107	1108	1109	1110
1111	1112	1113	1114	1115	1116
1117	1118	1119	1120	1121	1122
1123	1124	1125	1126	1127	1128
1129	1130	1131	1132	1133	1134
1135	1136	1137	1138	1139	1140
1141	1142	1143	1144	1145	1146
1147	1148	1149	1150	1151	1152
1153	1154	1155	1156	1157	1158
1159	1160	1161	1162	1163	1164
1165	1166	1167	1168	1169	1170
1171	1172	1173	1174	1175	1176
1177	1178	1179	1180	1181	1182
1183	1184	1185	1186	1187	1188
1189	1190	1191	1192	1193	1194
1195	1196	1197	1198	1199	1200
1201	1202	1203	1204	1205	1206
1207	1208	1209	1210	1211	1212
1213	1214	1215	1216	1217	1218
1219	1220	1221	1222	1223	1224
1225	1226	1227	1228	1229	1230
1231	1232	1233	1234	1235	1236
1237	1238	1239	1240	1241	1242
1243	1244	1245	1246	1247	1248
1249	1250	1251	1252	1253	1254
1255	1256	1257	1258	1259	1260
1261	1262	1263	1264	1265	1266
1267	1268	1269	1270	1271	1272
1273	1274	1275	1276	1277	1278
1279	1280	1281	1282	1283	1284
1285	1286	1287	1288	1289	1290
1291	1292	1293	1294	1295	1296
1297	1298	1299	1300	1301	1302
1303	1304	1305	1306	1307	1308
1309	1310	1311	1312	1313	1314
1315	1316	1317	1318	1319	1320
1321	1322	1323	1324	1325	1326
1327	1328	1329	1330	1331	1332
1333	1334				

Table 10. Conversion of 1948 Semi-Final Scores to Sigma Scale and T-Scores.

Score	d	d ²	Sigma Scale	T-Score
66	21	441	1.62	66.2
62	17	289	1.31	63.1
62	17	289	1.31	63.1
61	16	256	1.23	62.3
60	15	225	1.15	61.5
59	14	196	1.08	60.8
58	13	169	1.00	60.0
58	13	169	1.00	60.0
57	12	144	.92	59.2
56	11	121	.85	58.5
55	10	100	.77	57.7
54	9	81	.69	56.9
53	8	64	.62	56.2
53	8	64	.62	56.2
52	7	49	.54	55.4
50	5	25	.38	53.8
49	4	16	.31	53.1
44	-1	1	-.08	49.2
43	-2	4	-.15	48.5
43	-2	4	-.15	48.5
43	-2	4	-.15	48.5
42	-3	9	-.23	47.7
35	-10	100	-.77	42.3
34	-11	121	-.85	41.5
34	-11	121	-.85	41.5
34	-11	121	-.85	41.5
34	-11	121	-.85	41.5
33	-12	144	-.92	40.8
31	-14	196	-1.08	39.2
28	-17	289	-1.31	36.9

Date	Description	To	By	Amount
1917	Jan 1	Balance		100.00
	Feb 1	Interest		1.00
	Mar 1	Interest		1.00
	Apr 1	Interest		1.00
	May 1	Interest		1.00
	Jun 1	Interest		1.00
	Jul 1	Interest		1.00
	Aug 1	Interest		1.00
	Sep 1	Interest		1.00
	Oct 1	Interest		1.00
	Nov 1	Interest		1.00
	Dec 1	Interest		1.00
	Total			112.00

Table 10. (continued)

Score	d	d ²	Sigma Scale	T-Score
26	-19	361	-1.31	36.9
25	-20	400	-1.54	34.6
24	-21	441	-1.62	33.8
24	-21	441	-1.62	33.8
24	-21	441	-1.62	33.8
35 <u>1566</u>		35 <u>6017</u>		
44.74		171.91		

Mean = 45

Sigma = 13.1

Name	Age	Sex	Occupation	Address
John Doe	35	Male	Teacher	123 Main St.
Jane Smith	28	Female	Nurse	456 Oak Ave.
Robert Johnson	42	Male	Engineer	789 Pine Rd.
Emily White	22	Female	Student	321 Elm St.
Michael Brown	38	Male	Doctor	654 Maple Dr.
Sarah Green	30	Female	Artist	987 Cedar Ln.

Table 11. Conversion of 1949 Semi-Final Scores to Sigma Scale and T-Scores

Score	d	d ²	Sigma Scale	T-Score
67	28	784	1.9	69.0
65	26	676	1.77	67.7
62	23	529	1.56	65.6
59	20	400	1.36	63.6
58	19	361	1.29	62.9
55	16	256	1.09	60.9
52	13	169	.95	58.5
51	12	144	.86	58.2
50	11	121	.75	57.5
50	11	121	.75	57.5
45	6	36	.41	54.1
44	5	25	.34	53.4
44	5	25	.34	53.4
42	3	9	.20	52.0
42	3	9	.20	52.0
42	3	9	.20	52.0
39	0		0	50.
39	0		0	50.
38	-1	1	-.07	49.3
38	-1	1	-.07	49.3
37	-2	4	-.14	48.6
36	-3	9	-.20	48.0
36	-3	9	-.20	48.0
33	-6	36	-.41	45.9
33	-6	36	-.41	45.9
33	-6	36	-.41	45.9
31	-8	64	-.54	44.6
28	-11	121	-.75	42.5
26	-13	169	-.95	40.5
22	-17	289	-1.16	38.4

TABLE 1				
Year	1950	1955	1960	1965
1	100	100	100	100
2	100	100	100	100
3	100	100	100	100
4	100	100	100	100
5	100	100	100	100
6	100	100	100	100
7	100	100	100	100
8	100	100	100	100
9	100	100	100	100
10	100	100	100	100
11	100	100	100	100
12	100	100	100	100
13	100	100	100	100
14	100	100	100	100
15	100	100	100	100
16	100	100	100	100
17	100	100	100	100
18	100	100	100	100
19	100	100	100	100
20	100	100	100	100
21	100	100	100	100
22	100	100	100	100
23	100	100	100	100
24	100	100	100	100
25	100	100	100	100
26	100	100	100	100
27	100	100	100	100
28	100	100	100	100
29	100	100	100	100
30	100	100	100	100
31	100	100	100	100
32	100	100	100	100
33	100	100	100	100
34	100	100	100	100
35	100	100	100	100
36	100	100	100	100
37	100	100	100	100
38	100	100	100	100
39	100	100	100	100
40	100	100	100	100
41	100	100	100	100
42	100	100	100	100
43	100	100	100	100
44	100	100	100	100
45	100	100	100	100
46	100	100	100	100
47	100	100	100	100
48	100	100	100	100
49	100	100	100	100
50	100	100	100	100
51	100	100	100	100
52	100	100	100	100
53	100	100	100	100
54	100	100	100	100
55	100	100	100	100
56	100	100	100	100
57	100	100	100	100
58	100	100	100	100
59	100	100	100	100
60	100	100	100	100
61	100	100	100	100
62	100	100	100	100
63	100	100	100	100
64	100	100	100	100
65	100	100	100	100
66	100	100	100	100
67	100	100	100	100
68	100	100	100	100
69	100	100	100	100
70	100	100	100	100
71	100	100	100	100
72	100	100	100	100
73	100	100	100	100
74	100	100	100	100
75	100	100	100	100
76	100	100	100	100
77	100	100	100	100
78	100	100	100	100
79	100	100	100	100
80	100	100	100	100
81	100	100	100	100
82	100	100	100	100
83	100	100	100	100
84	100	100	100	100
85	100	100	100	100
86	100	100	100	100
87	100	100	100	100
88	100	100	100	100
89	100	100	100	100
90	100	100	100	100
91	100	100	100	100
92	100	100	100	100
93	100	100	100	100
94	100	100	100	100
95	100	100	100	100
96	100	100	100	100
97	100	100	100	100
98	100	100	100	100
99	100	100	100	100
100	100	100	100	100

Table 11. (continued)

Score	d	d ²	Sigma Scale	T-Score
17	-22	484	-1.50	35.0
15	-24	576	-1.63	33.7
14	-25	625	-1.7	33.0
13	-26	676	-1.77	32.3
12	-27	729	-1.84	31.6
35 1368		35 7539		
39.08		215.4		

Mean = 39

Sigma = 14.7

1. The first part of the document is a list of names and addresses.

2. The second part of the document is a list of names and addresses.

3. The third part of the document is a list of names and addresses.

4. The fourth part of the document is a list of names and addresses.

Table 12. Conversion of 1948 Final Scores to Sigma Scale and T-Scores

Score	d	d ²	Sigma Scale	T-Score
79	21	441	19.1	69.1
71	13	169	11.8	61.8
71	13	169	11.8	61.8
70	12	144	10.9	60.9
70	12	144	10.9	60.9
68	10	100	9.1	59.1
68	10	100	9.1	59.1
68	10	100	9.1	59.1
68	10	100	9.1	59.1
67	9	81	8.2	58.2
67	9	81	8.2	58.2
66	8	64	7.3	57.3
65	7	49	6.4	56.4
64	6	36	5.5	55.5
64	6	36	5.5	55.5
63	5	25	4.5	54.5
62	4	16	3.6	53.6
60	2	4	1.8	51.8
57	-1	1	.9	49.1
55	-3	9	-2.7	47.3
54	-4	16	-3.6	46.4
53	-5	25	-4.5	45.5
51	-7	49	-6.4	43.6
50	-8	64	-7.3	42.7
50	-8	64	-7.3	42.7
50	-8	64	-7.3	42.7
49	-9	81	-8.2	41.8
49	-9	81	-8.2	41.8
49	-9	81	-8.2	41.8
48	-10	100	-9.1	40.9

Table 12 (continued)

Score	d	d ²	Sigma Scale	T-Score
48	-10	100	-9.1	40.9
43	-15	225	-13.6	36.4
41	-17	289	-15.4	34.6
41	-17	289	-15.4	34.6
26	-32	1024	-29.0	21.0
35 <u>2025</u>		35 <u>4421</u>		
57.57		126.3		

Mean = 58

Sigma = 11.2

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Table 13. Conversion of 1949 Final Scores to Sigma Scale and T-Scores

Score	d	d ²	Sigma Scale	T-Score
73	18	324	16.4	66.4
72	17	289	15.5	65.5
71	16	256	14.5	64.5
68	13	169	12.8	61.8
67	12	144	10.9	60.9
67	12	144	10.9	60.9
64	9	81	8.2	58.2
63	8	64	7.3	57.3
63	8	64	7.3	57.3
61	6	36	5.5	55.5
61	6	36	5.5	55.5
61	6	36	5.5	55.5
58	3	9	2.7	52.7
58	3	9	2.7	52.7
57	2	4	1.8	51.8
56	1	1	.9	50.9
56	1	1	.9	50.9
55	0		0	50.0
55	0		0	50.0
55	0		0	50.0
54	-1	1	-1.9	49.1
53	-2	4	-1.8	48.2
53	-2	4	-1.8	48.2
53	-2	4	-1.8	48.2
53	-2	4	-1.8	48.2
51	-4	16	-3.6	46.4
50	-5	25	-4.5	45.6
49	-6	36	-5.5	44.5
49	-6	36	-5.5	44.5
48	-7	49	-6.4	43.6

Table 13. (continued)

Score	d	d^2	Sigma Scale	T-Score
43	-12	144	-10.9	39.1
35	-20	400	-18.2	31.8
35	-20	400	-18.2	31.8
29	-26	676	-23.6	26.4
23	-32	1024	-29.1	20.9
35 <u>1919</u>		35 <u>4490</u>		
54.82		128.28		

Mean = 55

Sigma 11.3

Table 14. Significance of the Differences between the Means of the 1948 and 1949 Groups on the Semi-Final and Final Test Scores

A. Semi-Final Test

1948 35 Students Mean = 45 Sigma = 13.1

1949 35 Students Mean = 39 Sigma = 14.7

$$\text{Sigma}_{M_{48}} = \frac{13.1}{\sqrt{35}} = 2.22 \qquad \text{Sigma}_{M_{49}} = \frac{14.7}{\sqrt{35}} = 2.48$$

$$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{48}} - \text{Sigma}_{M_{49}} = \sqrt{2.22^2 + 2.48^2}$$

$$\text{Sigma}_D = 3.3$$

$$\frac{D}{\text{Sigma}_D} = \frac{6}{3.3} = 1.8^*$$

*From Garrett Table 34, p. 213. Chances are 96 in 100

B. Final Test

1948 35 Students Mean = 58 Sigma = 11.2

1949 35 Students Mean = 55 Sigma = 11.3

$$\text{Sigma}_{M_{48}} = \frac{11.2}{\sqrt{35}} = 1.90 \qquad \text{Sigma}_{M_{49}} = \frac{11.3}{\sqrt{35}} = 1.92$$

$$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{48}} - \text{Sigma}_{M_{49}} = \sqrt{1.90^2 + 1.92^2}$$

$$\text{Sigma}_D = 2.7$$

$$\frac{D}{\text{Sigma}_D} = \frac{3.0}{2.7} = 1.1^*$$

*From Garrett Table 34, p. 213. Chances are 86 in 100

THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

PHYSICS 341

LECTURE 1

1.1. THE CLASSICAL LIMIT

1.2. THE QUANTUM LIMIT

1.3. THE CORRESPONDENCE PRINCIPLE

1.4. THE UNCERTAINTY PRINCIPLE

1.5. THE WAVE FUNCTION

1.6. THE SCHRÖDINGER EQUATION

1.7. THE HAMILTONIAN

1.8. THE EIGENVALUE PROBLEM

1.9. THE ENERGY LEVELS

1.10. THE PROBABILITY DENSITY

1.11. THE EXPECTED VALUE

1.12. THE VARIATIONAL PRINCIPLE

1.13. THE PERTURBATION THEORY

1.14. THE ADIABATIC APPROXIMATION

1.15. THE TUNNELING EFFECT

1.16. THE SPIN

1.17. THE PAULI EXCLUSION PRINCIPLE

1.18. THE FERMI-Dirac STATISTICS

1.19. THE BOSE-EINSTEIN STATISTICS

1.20. THE SUPERCONDUCTIVITY

1.21. THE SUPERFLUIDITY

1.22. THE QUANTUM HALL EFFECT

1.23. THE QUANTUM COHERENCE

1.24. THE QUANTUM ENTANGLEMENT

1.25. THE QUANTUM TELEPORTATION

1.26. THE QUANTUM CRYPTOGRAPHY

1.27. THE QUANTUM COMPUTING

1.28. THE QUANTUM COMMUNICATIONS

1.29. THE QUANTUM NETWORKS

1.30. THE QUANTUM INFORMATION THEORY

1.31. THE QUANTUM MECHANICS

1.32. THE QUANTUM FIELD THEORY

1.33. THE QUANTUM GRAVITY

1.34. THE QUANTUM COSMOLOGY

1.35. THE QUANTUM BLACK HOLES

1.36. THE QUANTUM DARK MATTER

1.37. THE QUANTUM DARK ENERGY

1.38. THE QUANTUM UNIFICATION

1.39. THE QUANTUM FOUNDATIONS

1.40. THE QUANTUM INTERPRETATIONS

1.41. THE QUANTUM MEASUREMENT

1.42. THE QUANTUM DECOHERENCE

1.43. THE QUANTUM ZENO EFFECT

1.44. THE QUANTUM CHAOS

1.45. THE QUANTUM THERMODYNAMICS

1.46. THE QUANTUM STATISTICS

1.47. THE QUANTUM OPTICS

1.48. THE QUANTUM ELECTRODYNAMICS

1.49. THE QUANTUM CHROMODYNAMICS

1.50. THE QUANTUM WEAK INTERACTIONS

1.51. THE QUANTUM STRONG INTERACTIONS

1.52. THE QUANTUM STANDARD MODEL

1.53. THE QUANTUM BEYOND STANDARD MODEL

1.54. THE QUANTUM COSMOLOGICAL MODELS

1.55. THE QUANTUM GRAVITATIONAL MODELS

1.56. THE QUANTUM UNIFIED THEORIES

1.57. THE QUANTUM FUNDAMENTALS

1.58. THE QUANTUM FRONTIERS

1.59. THE QUANTUM FUTURE

1.60. THE QUANTUM CONCLUSION

PHYSICS 341

LECTURE 2

2.1. THE CLASSICAL LIMIT

2.2. THE QUANTUM LIMIT

2.3. THE CORRESPONDENCE PRINCIPLE

2.4. THE UNCERTAINTY PRINCIPLE

2.5. THE WAVE FUNCTION

2.6. THE SCHRÖDINGER EQUATION

2.7. THE HAMILTONIAN

2.8. THE EIGENVALUE PROBLEM

2.9. THE ENERGY LEVELS

2.10. THE PROBABILITY DENSITY

2.11. THE EXPECTED VALUE

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2.47. THE QUANTUM OPTICS

2.48. THE QUANTUM ELECTRODYNAMICS

2.49. THE QUANTUM CHROMODYNAMICS

2.50. THE QUANTUM WEAK INTERACTIONS

2.51. THE QUANTUM STRONG INTERACTIONS

2.52. THE QUANTUM STANDARD MODEL

2.53. THE QUANTUM BEYOND STANDARD MODEL

2.54. THE QUANTUM COSMOLOGICAL MODELS

2.55. THE QUANTUM GRAVITATIONAL MODELS

2.56. THE QUANTUM UNIFIED THEORIES

2.57. THE QUANTUM FUNDAMENTALS

2.58. THE QUANTUM FRONTIERS

2.59. THE QUANTUM FUTURE

2.60. THE QUANTUM CONCLUSION

Table 15. Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the Groups on Semi-Final Tests

Upper Half of 1948 Group			Upper Half of 1949 Group		
Score	d	d ²	Score	d	d ²
66	15	225	67	18	324
62	11	121	65	16	256
62	11	121	62	13	169
61	10	100	59	10	100
60	9	81	58	9	81
59	8	64	44	6	36
56	5	25	52	3	9
54	3	9	51	2	4
53	2	4	45	-4	16
52	1	1	44	-5	25
52	-1	1	44	-5	25
44	-7	49	42	-7	49
43	-8	64	42	-7	49
43	-8	64	37	-12	144
42	-9	81	36	-13	169
34	-17	289	36	-13	169
34	-17	289	33	-16	256
17 <u>1875</u>		17 <u>1588.0</u>	17 <u>1828</u>		17 <u>1881</u>
51		93.41	48.7		110.64

Mean = 51

Sigma = 9.7

Mean = 49

Sigma = 10.5

Sigma_D of Upper Halves

$$M_{48} - M_{49} = 51 - 49 = 2.0$$

$$\text{Sigma}_{M_{48}} = \frac{9.7}{\sqrt{17}} = 2.34$$

$$\text{Sigma}_{M_{49}} = \frac{10.5}{\sqrt{17}} = 2.55$$

$$\text{Sigma}_D = \sqrt{2.34^2 + 2.55^2}$$

$$\text{Sigma}_D = 3.5$$

$$\frac{D}{\text{Sigma}_D} = \frac{2.0}{3.5} = .58$$

From Garrett Table 34 at .60

Chances are 73 in 100

Table 16. Comparison of Scores and Significance of the difference between the Means for the Lower Halves of the Groups on Semi-Final Tests

Lower Half of 1948 Group			Lower Half of 1949 Group		
Score	d	d ²	Score	d	d ²
58	22	441	50	20	400
58	21	441	50	20	400
55	18	324	42	12	144
53	16	256	39	9	81
49	12	144	39	9	81
43	6	36	38	8	64
35	-2	4	38	8	64
34	-3	9	33	3	9
34	-3	9	33	3	9
33	-4	16	31	1	1
31	-6	36	28	-2	4
28	-9	81	22	-8	64
26	-11	121	17	-13	169
25	-12	144	15	-15	225
24	-13	169	14	-16	256
24	-13	169	13	-17	289
24	-13	169	12	-18	324
17 <u>634</u>		17 <u>2569</u>	17 <u>514</u>		17 <u>2584</u>
37.2		151.1	30.2		152

Mean = 37

Sigma = 12.3

Mean = 30

Sigma = 12.3

Sigma_D of Lower Halves

$$M_{48} - M_{49} = 37 - 30 = 7.0$$

$$\text{Sigma}_{M_{48}} = \frac{12.3}{\sqrt{17}} = 2.98$$

$$\text{Sigma}_{M_{49}} = \frac{12.3}{\sqrt{17}} = 2.90$$

$$\text{Sigma}_D = \sqrt{2.98^2 + 2.90^2}$$

$$\text{Sigma}_D = 4.2$$

$$\frac{D}{\text{Sigma}_D} = \frac{7.0}{4.2} = 1.6$$

From Garrett Table 34 at 1.60

Chances are 94 in 100

Table 17. Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the groups on Final Tests

Upper Half of 1948 Group			Upper Half of 1949 Group		
Score	d	d ²	Score	d	d ²
79	16	256	73	12	144
71	8	64	72	11	121
71	8	64	71	10	100
70	7	49	68	6	36
70	7	49	67	5	25
68	5	25	67	5	25
68	5	25	64	2	4
68	5	25	63	1	1
67	4	16	61	-1	1
66	3	9	31	-1	1
65	2	4	58	-4	16
64	1	1	58	-4	16
54	-9	81	56	-6	36
50	-13	169	55	-6	36
50	-13	169	53	-9	81
49	-14	199	51	-11	121
41	-22	484	50	-12	144
17 <u>1071</u>		17 <u>1689</u>	17 <u>1048</u>		17 <u>9080</u>
63		99.3	61.6		53.41

Mean = 63

Sigma = 10

Mean = 62

Sigma = 7.33

Sigma_D of Upper Halves

$$M_{48} - M_{49} = 63 - 62 = 1.0$$

$$\text{Sigma}_{M_{48}} = \frac{10}{\sqrt{17}} = 2.41$$

$$\text{Sigma}_{M_{49}} = \frac{7.3}{\sqrt{17}} = 1.78$$

$$\text{Sigma}_D = \sqrt{2.41^2 + 1.78^2}$$

$$\text{Sigma}_D = 3.0$$

$$\frac{D}{\text{Sigma}_D} = \frac{1.0}{3.0} = .33$$

From Garrett Table 34 at .35

Chances are 64 in 100

THE UNIVERSITY OF CHICAGO

DEPARTMENT OF CHEMISTRY

PHYSICAL CHEMISTRY

LECTURE NOTES

BY

PROFESSOR

JOHN D. COLEMAN

CHICAGO, ILLINOIS

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Table 18. Comparison of Scores and Significance of the Difference between the Means for the Lower Halves of the Groups on Final Tests

Lower Half of 1948 Group			Lower Half of 1949 Group		
Score	d	d ²	Score	d	d ²
67	15	225	63	14	196
64	12	144	61	12	144
63	11	121	57	8	64
62	10	100	56	7	49
60	8	64	55	6	36
57	5	25	35	6	36
55	3	9	54	5	25
53	1	1	53	4	16
51	-1	1	53	4	16
50	-2	4	53	4	16
49	-3	9	49	0	0
49	-3	9	49	0	0
48	-4	16	48	-1	1
48	-4	16	43	-6	36
43	-9	81	35	-14	196
41	-11	121	35	-14	196
26	-26	676	29	-20	400
17 <u>8860</u>		17 <u>1622</u>	17 <u>8280</u>		17 <u>1427</u>
52.1		95.4	48.7		83.94
Mean = 52		Sigma = 9.75	Mean = 49		Sigma = 9.15

Sigma_D of Lower Halves

$$M_{48} - M_{49} = 52 - 49 = 3.0$$

$$\text{Sigma}_{M_{48}} = \frac{9.75}{\sqrt{17}} = 2.36$$

$$\text{Sigma}_{M_{49}} = \frac{9.15}{\sqrt{17}} = 2.22$$

$$\text{Sigma}_D = \sqrt{2.36^2 + 2.22^2}$$

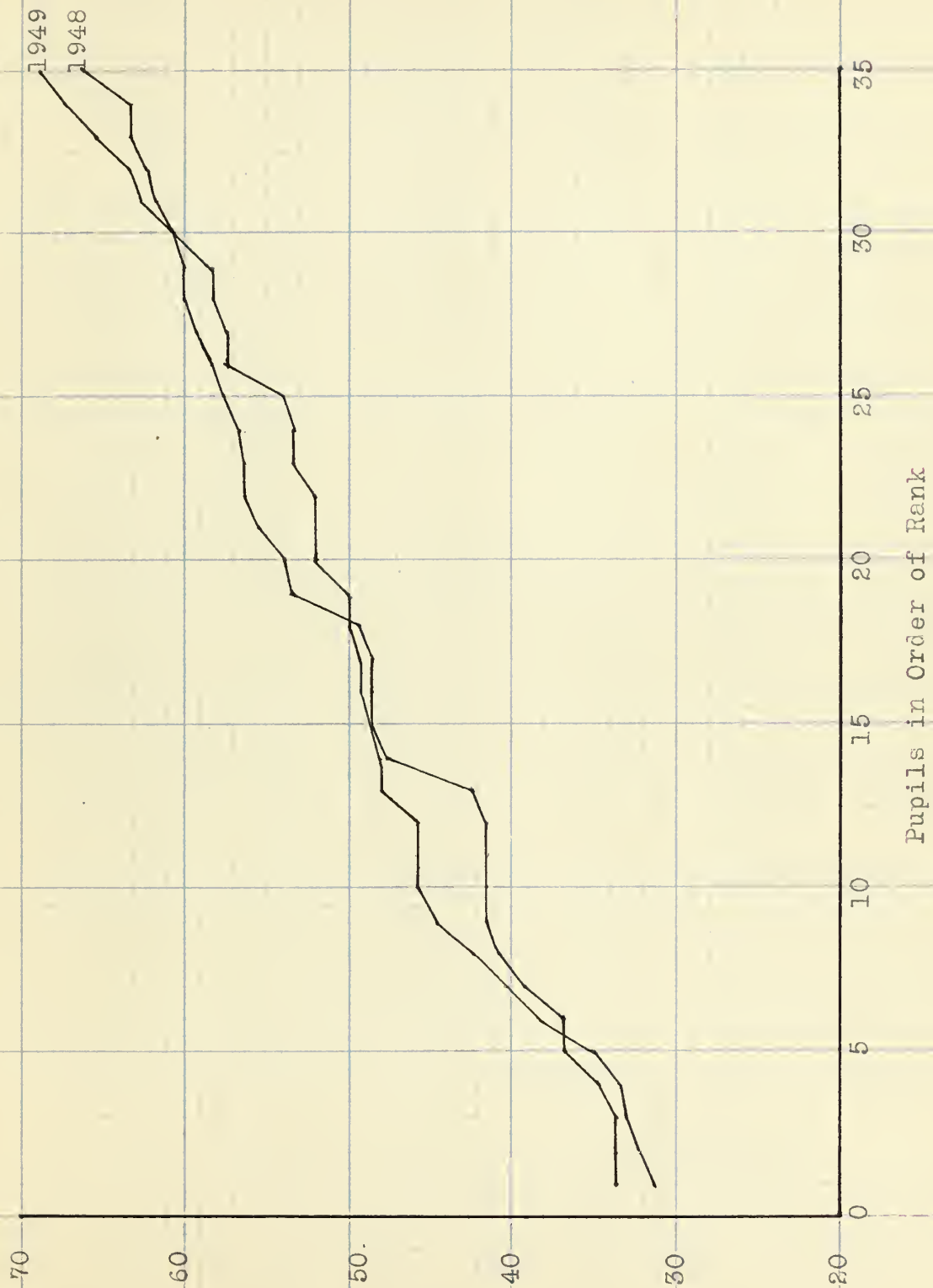
$$\text{Sigma}_D = 3.2$$

$$\frac{D}{\text{Sigma}_D} = \frac{3.0}{3.2} = .93$$

From Garrett Table 34 at .95

Chances are 83 in 100

Table 19. Graphs of Achievement on Semi-Final Tests for 1948 and 1949 Groups



T-Scores - 1948 and 1949 Semi-Final Tests

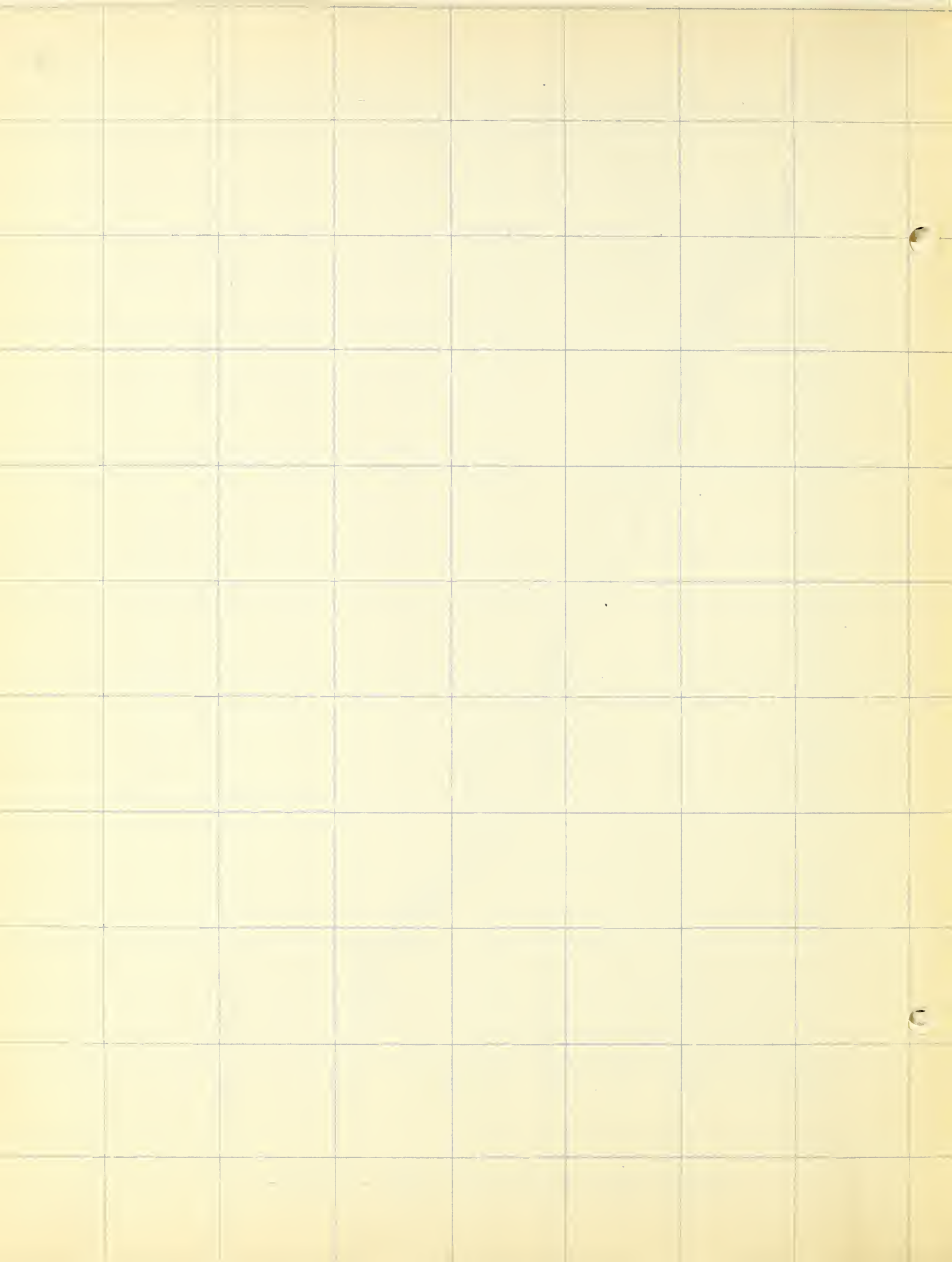
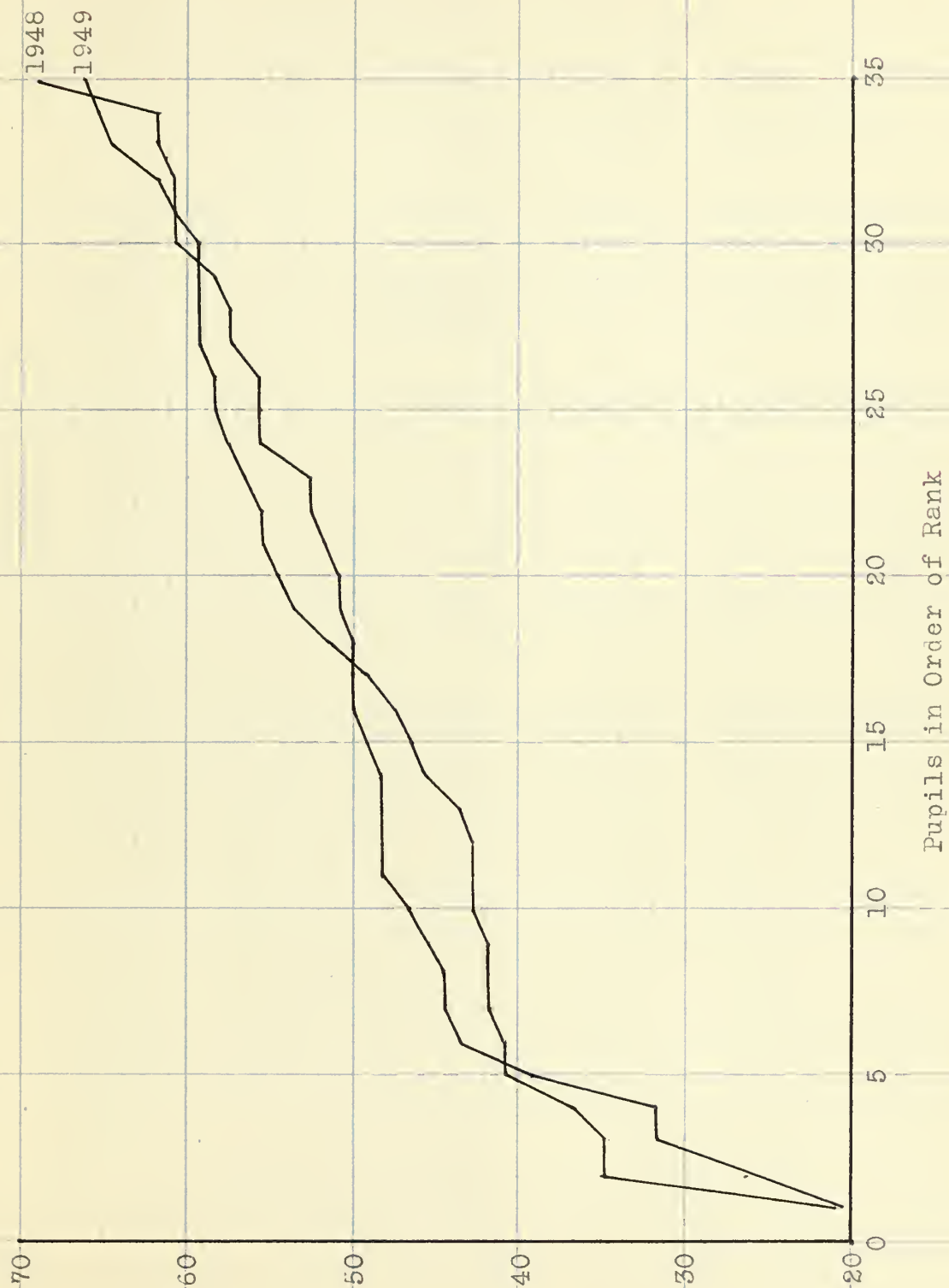
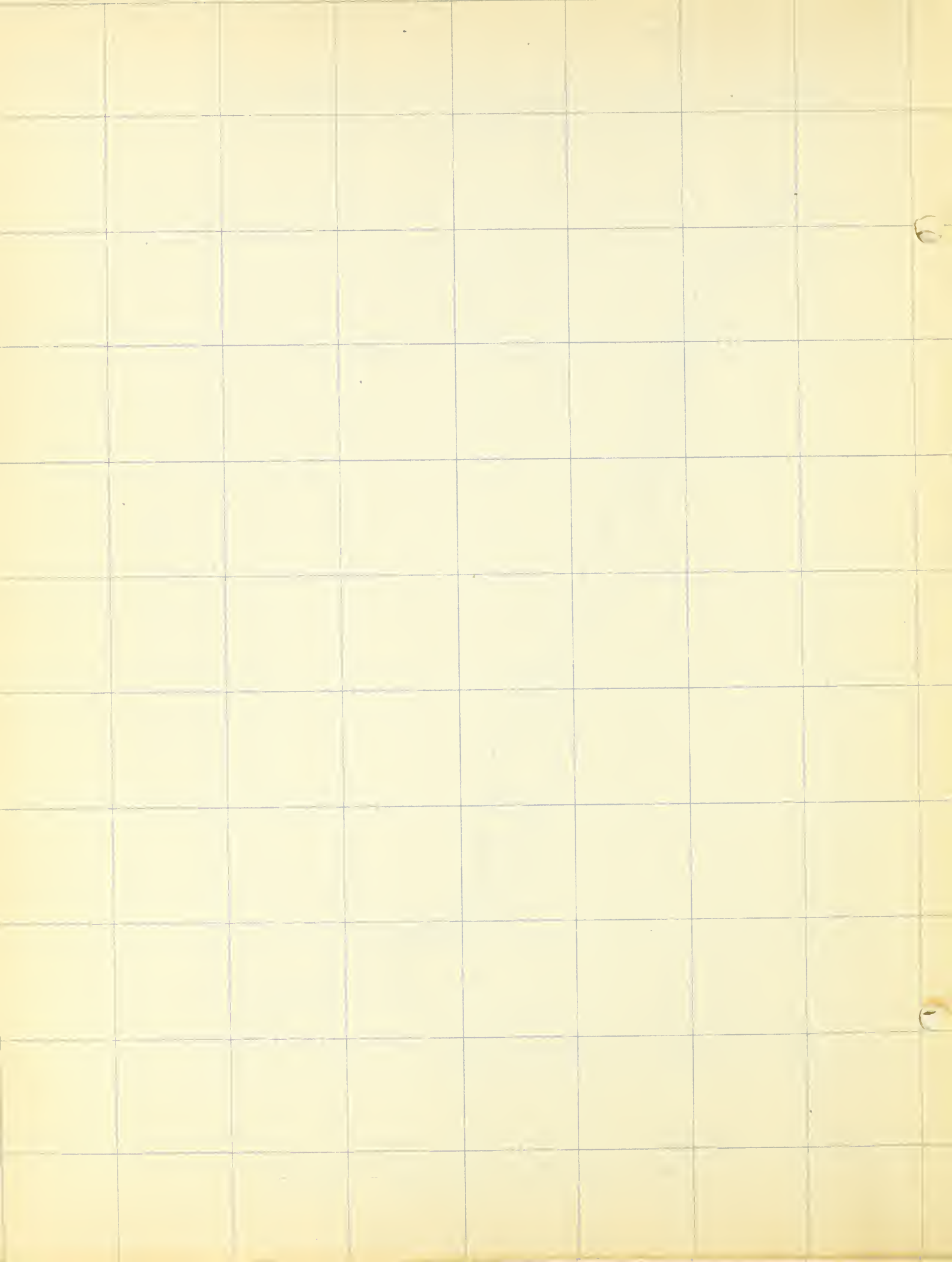


Table 20. Graphs of Achievement on Final Tests for 1948 and 1949-Groups

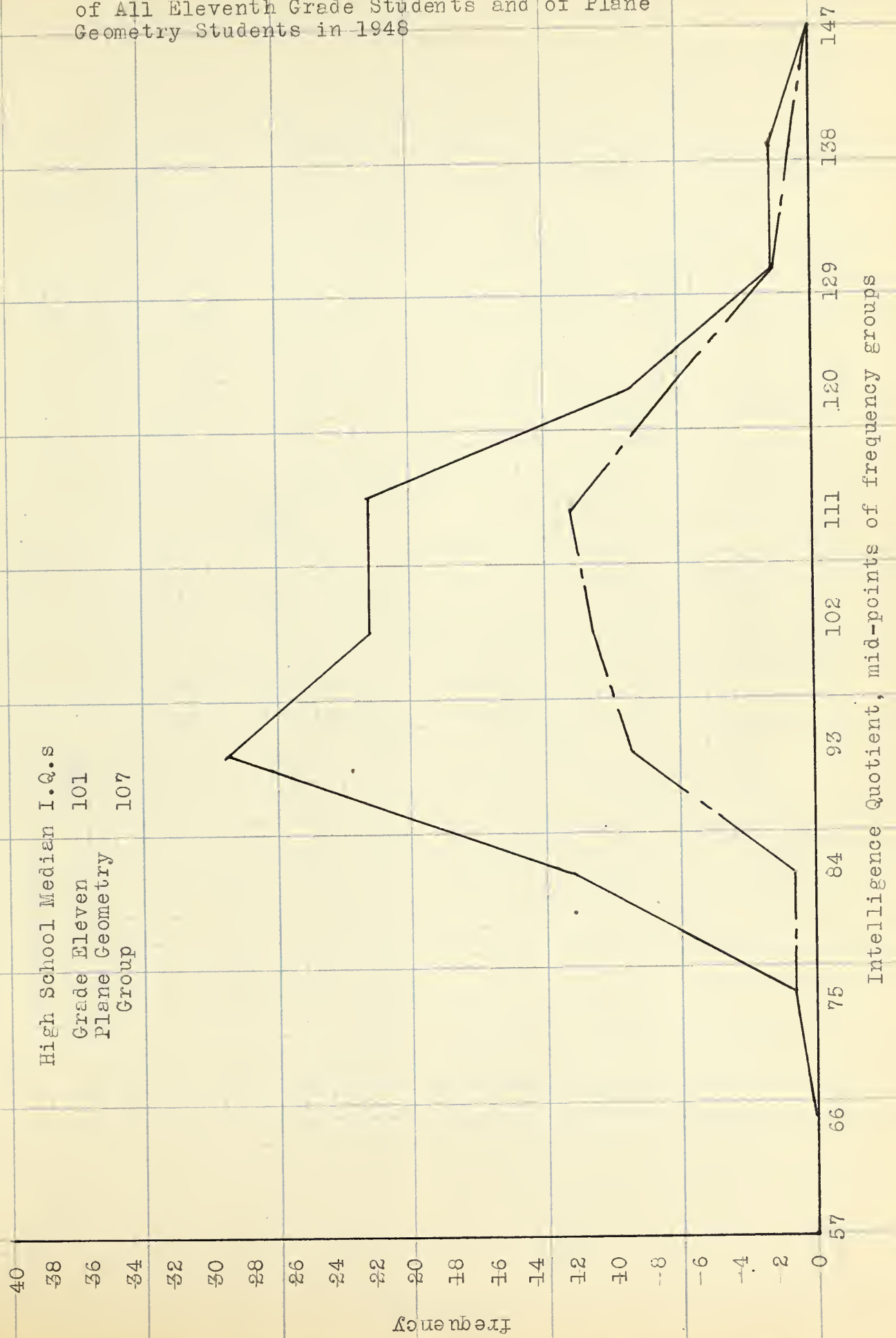


T-Scores - 1948 and 1949 Final Tests



APPENDIX B

Table 1. Graphs of Intelligence Quotient versus Frequency of All Eleventh Grade Students and of Plane Geometry Students in 1948



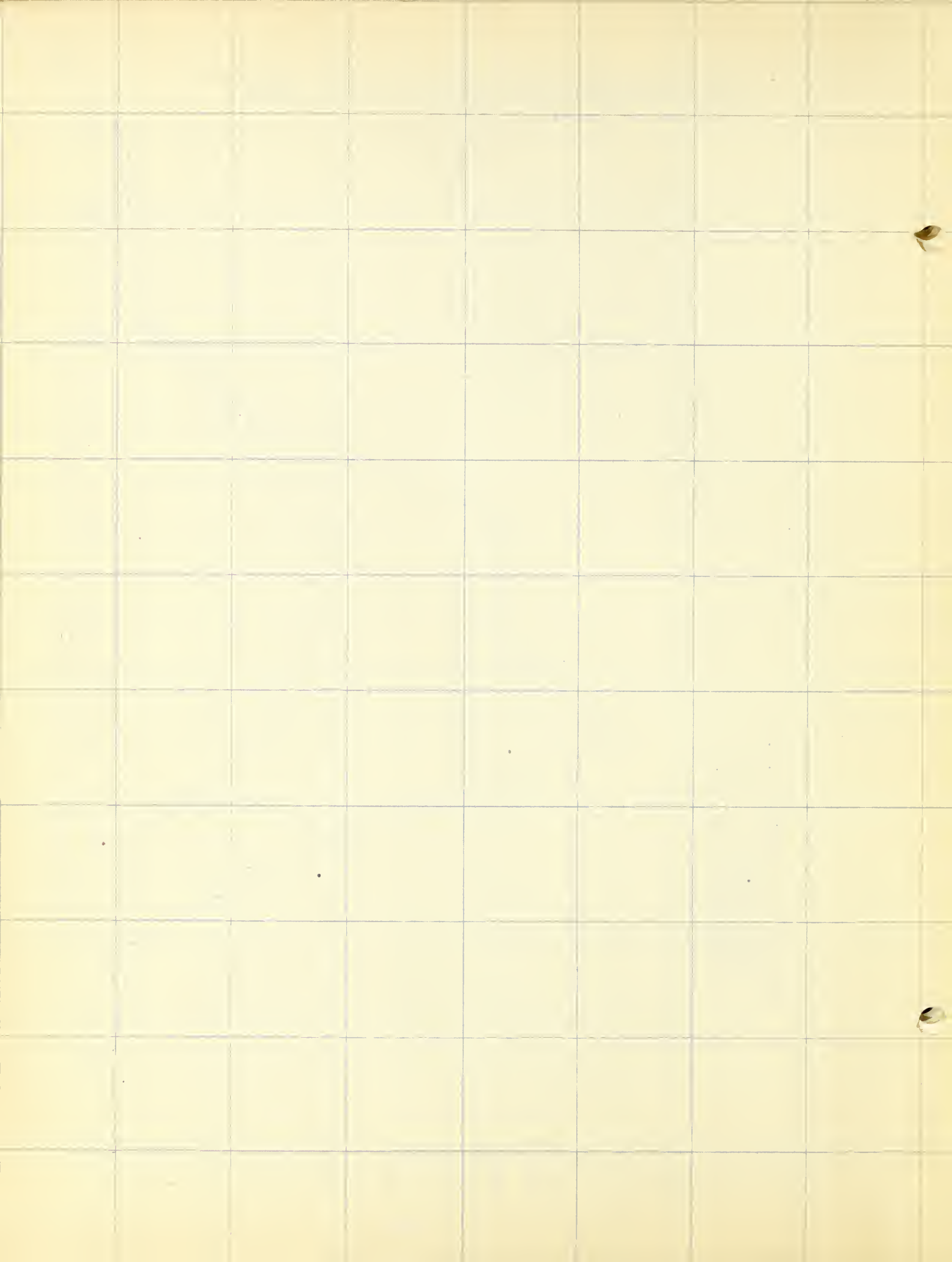
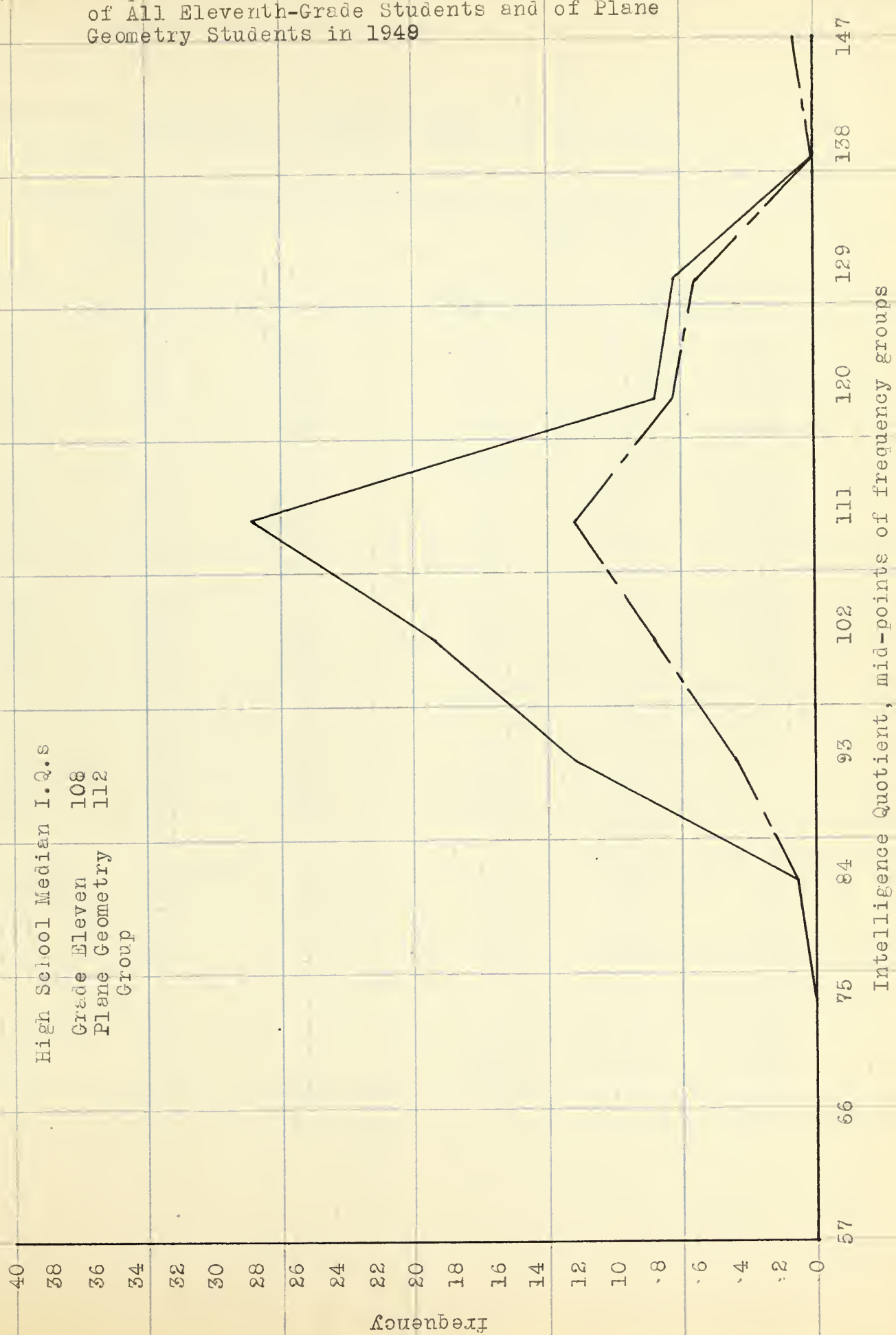


Table 2. Graphs of Intelligence Quotient versus Frequency of All Eleventh-Grade Students and of Plane Geometry Students in 1949

74



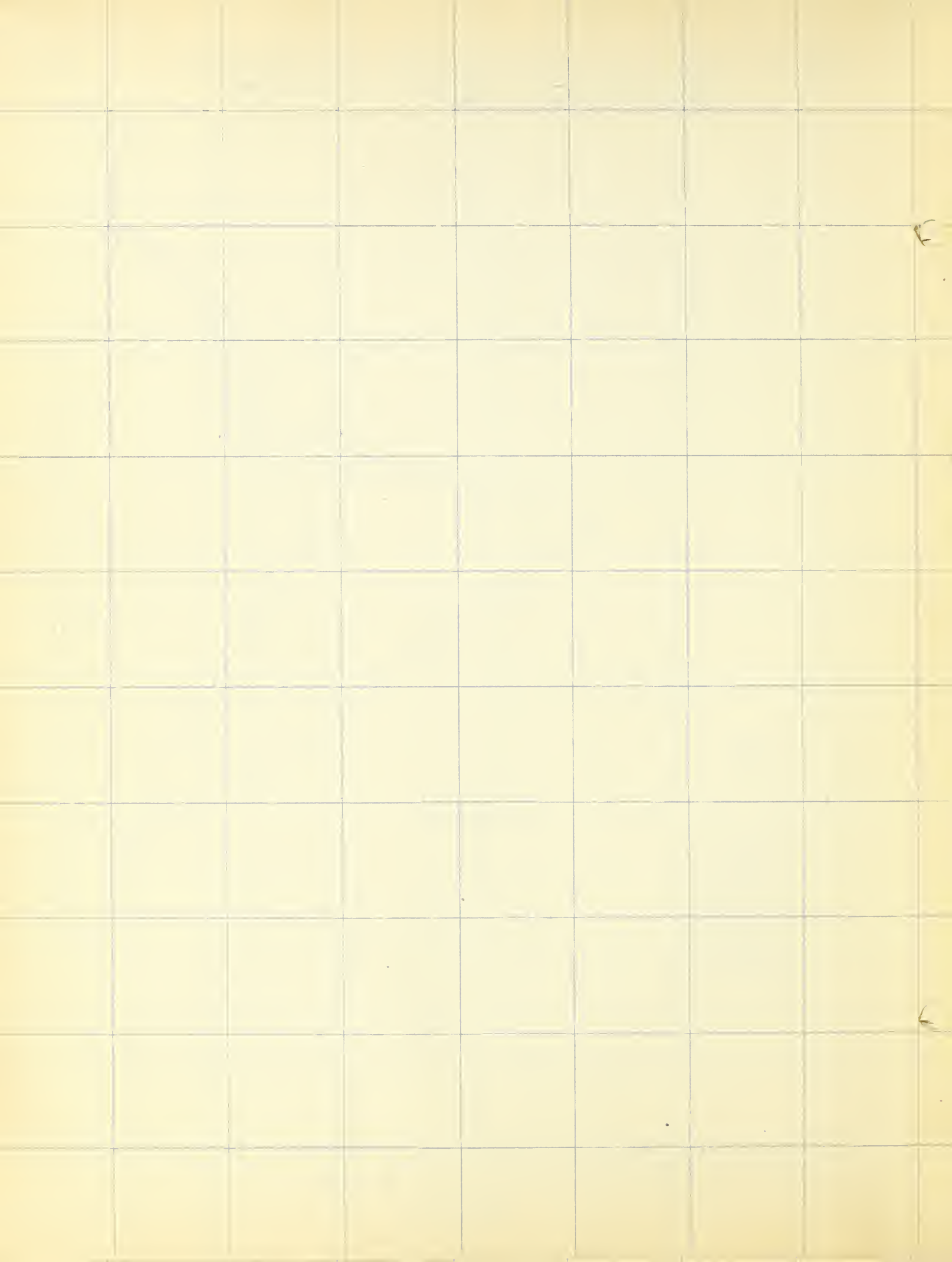


Table 3. Data Used in Calculating Mean Intelligence Quotient and Sigma of Unequated Groups

<u>1948 Groups</u>			<u>1949 Groups</u>		
<u>IQ</u>	<u>d</u>	<u>d²</u>	<u>IQ</u>	<u>d</u>	<u>d²</u>
140	35	1225	143	31	961
128	23	529	132	20	400
128	23	529	129	17	289
124	19	361	126	14	196
122	17	289	125	13	169
121	16	256	125	13	169
120	15	225	125	13	169
117	12	144	124	12	144
117	12	144	124	12	144
116	11	121	123	11	121
115	10	100	122	10	100
113	8	64	119	7	49
112	7	49	118	6	36
111	6	36	117	5	25
111	6	36	115	3	9
110	5	25	113	1	1
110	5	25	112	0	
110	5	25	112	0	
109	4	16	112	0	
109	4	16	112	0	
108	3	9	111	-1	1
107	2	4	110	-2	4
106	1	1	109	-3	9
106	1	1	109	-3	9
104	-1	1	108	-4	16
104	-1	1	107	-5	25
104	-1	1	106	-6	36
102	-3	9	106	-6	36
102	-3	9	106	-6	36
100	-5	25	102	-10	100

Table 3. (continued)

<u>1948 Groups</u>			<u>1949 Groups</u>		
IQ	d	d ²	IQ	d	d ²
98	-7	49	102	-10	100
98	-7	49	100	-12	144
98	-7	49	98	-14	196
97	-8	64	98	-14	196
97	-8	64	97	-15	225
97	-8	64	97	-15	225
97	-8	64	95	-17	289
96	-9	81	95	-17	289
95	-10	100	86	-26	576
94	-11	121			
92	-13	169			
90	-15	225			
88	-17	289			
78	-27	729			
<u>4601</u>		<u>6393</u>	<u>4370</u>		<u>5594</u>

$$\text{Mean} = 44 \frac{104.56}{4601.00}$$

$$\text{Mean} = 105$$

$$\text{Sigma} = \frac{\sqrt{145.29}}{44} = 12.05$$

$$\text{Sigma} = 12.1$$

$$\text{Mean} = 39 \frac{112.05}{4370.00}$$

$$\text{Mean} = 112$$

$$\text{Sigma} = \frac{\sqrt{143.43}}{44} = 11.98$$

$$\text{Sigma} = 12.0$$

Table 4. Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups - First Trial

<u>1948 Groups</u>			<u>1949 Groups</u>		
IQ	d	d ²	IQ	d	d ²
140	34	1156	129	15	225
128	22	484	126	12	144
128	22	484	125	11	121
124	18	324	125	11	121
122	16	256	125	11	121
121	15	225	124	10	100
120	14	196	124	10	100
117	11	121	123	9	81
117	11	121	122	8	64
116	10	100	119	5	25
115	9	81	118	4	16
113	7	49	117	3	9
112	6	36	115	1	1
111	5	25	113	-1	1
111	5	25	112	-2	4
110	4	16	112	-2	4
110	4	16	112	-2	4
110	4	16	112	-2	4
109	3	9	111	-3	9
109	3	9	110	-4	16
108	2	4	109	-5	25
107	1	1	109	-5	25
106	0		108	-6	36
106	0		107	-7	49
104	-2	4	106	-8	64
104	-2	4	106	-8	64
104	-2	4	106	-8	64
102	-4	16	102	-12	144
102	-4	16	102	-12	144
100	-6	36	100	-14	196

Table 4. (continued)

<u>1948 Groups</u>			<u>1949 Groups</u>		
IQ	d	d ²	IQ	d	d ²
98	-8	64	98	-16	256
98	-8	64	98	-16	256
98	-8	64	97	-17	289
97	-9	81	97	-17	289
97	-9	81	95	-19	361
97	-9	81	95	-19	361
97	-9	81	86	-28	784
96	-10	100			
95	-11	121			
94	-12	144			
<u>4253</u>		<u>4715</u>	<u>4207</u>		<u>4577</u>

$$\text{Mean} = 40 \quad \frac{106.32}{4253.00}$$

$$\text{Mean} = 106$$

$$\text{Sigma} = \frac{\sqrt{117.87}}{40} = 10.86$$

$$\text{Sigma} = 10.9$$

$$\text{Mean} = 37 \quad \frac{113.64}{4207.00}$$

$$\text{Mean} = 114$$

$$\text{Sigma} = \frac{\sqrt{123.7}}{37} = 11.14$$

$$\text{Sigma} = 11.1$$

1900		1901		1902		1903		1904		1905		1906		1907		1908		1909		1910		1911		1912		1913		1914		1915		1916		1917		1918		1919		1920		1921		1922		1923		1924		1925		1926		1927		1928		1929		1930		1931		1932		1933		1934		1935		1936		1937		1938		1939		1940		1941		1942		1943		1944		1945		1946		1947		1948		1949		1950		1951		1952		1953		1954		1955		1956		1957		1958		1959		1960		1961		1962		1963		1964		1965		1966		1967		1968		1969		1970		1971		1972		1973		1974		1975		1976		1977		1978		1979		1980		1981		1982		1983		1984		1985		1986		1987		1988		1989		1990		1991		1992		1993		1994		1995		1996		1997		1998		1999		2000		2001		2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012		2013		2014		2015		2016		2017		2018		2019		2020		2021		2022		2023		2024		2025		2026		2027		2028		2029		2030		2031		2032		2033		2034		2035		2036		2037		2038		2039		2040		2041		2042		2043		2044		2045		2046		2047		2048		2049		2050		2051		2052		2053		2054		2055		2056		2057		2058		2059		2060		2061		2062		2063		2064		2065		2066		2067		2068		2069		2070		2071		2072		2073		2074		2075		2076		2077		2078		2079		2080		2081		2082		2083		2084		2085		2086		2087		2088		2089		2090		2091		2092		2093		2094		2095		2096		2097		2098		2099		2100		2101		2102		2103		2104		2105		2106		2107		2108		2109		2110		2111		2112		2113		2114		2115		2116		2117		2118		2119		2120		2121		2122		2123		2124		2125		2126		2127		2128		2129		2130		2131		2132		2133		2134		2135		2136		2137		2138		2139		2140		2141		2142		2143		2144		2145		2146		2147		2148		2149		2150		2151		2152		2153		2154		2155		2156		2157		2158		2159		2160		2161		2162		2163		2164		2165		2166		2167		2168		2169		2170		2171		2172		2173		2174		2175		2176		2177		2178		2179		2180		2181		2182		2183		2184		2185		2186		2187		2188		2189		2190		2191		2192		2193		2194		2195		2196		2197		2198		2199		2200		2201		2202		2203		2204		2205		2206		2207		2208		2209		2210		2211		2212		2213		2214		2215		2216		2217		2218		2219		2220		2221		2222		2223		2224		2225		2226		2227		2228		2229		2230		2231		2232		2233		2234		2235		2236		2237		2238		2239		2240		2241		2242		2243		2244		2245		2246		2247		2248		2249		2250		2251		2252		2253		2254		2255		2256		2257		2258		2259		2260		2261		2262		2263		2264		2265		2266		2267		2268		2269		2270		2271		2272		2273		2274		2275		2276		2277		2278		2279		2280		2281		2282		2283		2284		2285		2286		2287		2288		2289		2290		2291		2292		2293		2294		2295		2296		2297		2298		2299		2300		2301		2302		2303		2304		2305		2306		2307		2308		2309		2310		2311		2312		2313		2314		2315		2316		2317		2318		2319		2320		2321		2322		2323		2324		2325		2326		2327		2328		2329		2330		2331		2332		2333		2334		2335		2336		2337		2338		2339		2340		2341		2342		2343		2344		2345		2346		2347		2348		2349		2350		2351		2352		2353		2354		2355		2356		2357		2358		2359		2360		2361		2362		2363		2364		2365		2366		2367		2368		2369		2370		2371		2372		2373		2374		2375		2376		2377		2378		2379		2380		2381		2382		2383		2384		2385		2386		2387		2388		2389		2390		2391		2392		2393		2394		2395		2396		2397		2398		2399		2400		2401		2402		2403		2404		2405		2406		2407		2408		2409		2410		2411		2412		2413		2414		2415		2416		2417		2418		2419		2420		2421		2422		2423		2424		2425		2426		2427		2428		2429		2430		2431		2432		2433		2434		2435		2436		2437		2438		2439		2440		2441		2442		2443		2444		2445		2446		2447		2448		2449		2450		2451		2452		2453		2454		2455		2456		2457		2458		2459		2460		2461		2462		2463		2464		2465		2466		2467		2468		2469		2470		2471		2472		2473		2474		2475		2476		2477		2478		2479		2480		2481		2482		2483		2484		2485		2486		2487		2488		2489		2490		2491		2492		2493		2494		2495		2496		2497		2498		2499		2500		2501		2502		2503		2504		2505		2506		2507		2508		2509		2510		2511		2512		2513		2514		2515		2516		2517		2518		2519		2520		2521		2522		2523		2524		2525		2526		2527		2528		2529		2530		2531		2532		2533		2534		2535		2536		2537		2538		2539		2540		2541		2542		2543		2544		2545		2546		2547		2548		2549		2550		2551		2552		2553		2554		2555		2556		2557		2558		2559		2560		2561		2562		2563		2564		2565		2566		2567		2568		2569		2570		2571		2572		2573		2574		2575		2576		2577		2578		2579		2580		2581		2582		2583		2584		2585		2586		2587		2588		2589		2590		2591		2592		2593		2594		2595		2596		2597		2598		2599		2600		2601		2602		2603		2604		2605		2606		2607		2608		2609		2610		2611		2612		2613		2614		2615		2616		2617		2618		2619		2620		2621		2622		2623		2624		2625		2626		2627		2628		2629		2630		2631		2632		2633		2634		2635		2636		2637		2638		2639		2640		2641		2642		2643		2644		2645		2646		2647		2648		2649		2650		2651		2652		2653		2654		2655		2656		2657		2658		2659		2660		2661		2662		2663		2664		2665		2666		2667		2668		2669		2670		2671		2672		2673		2674		2675		2676		2677		2678		2679		2680		2681		2682		2683		2684		2685		2686		2687		2688		2689		2690		2691		2692		2693		2694		2695		2696		2697		2698		2699		2700		2701		2702		2703		2704		2705		2706		2707		2708		2709		2710		2711		2712		2713		2714		2715		2716		2717		2718		2719		2720		2721		2722		2723		2724		2725		2726		2727		2728		2729		2730		2731		2732		27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Table 5. Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups

<u>1948 Groups</u>			<u>1949 Groups</u>		
IQ	d	d ²	IQ	d	d ²
140	32	1024	125	15	225
128	20	400	125	15	225
128	20	400	125	15	225
124	16	256	124	14	196
122	14	196	124	14	196
121	13	169	123	13	169
120	12	144	122	12	144
117	9	81	119	9	81
117	9	81	118	8	64
116	8	64	117	7	49
115	7	49	115	5	25
113	5	25	113	3	9
112	4	16	112	2	4
111	3	9	112	2	4
111	3	9	112	2	4
110	2	4	112	2	4
110	2	4	111	1	1
110	2	4	110	0	
109	1	1	109	-1	1
109	1	1	109	-1	1
108	0		108	-2	4
107	-1	1	107	-3	9
106	-2	4	106	-4	16
106	-2	4	106	-4	16
104	-4	16	106	-4	16
104	-4	16	102	-8	64
104	-4	16	102	-8	64
102	-6	36	100	-10	100
102	-6	36	98	-12	144
100	-8	64	98	-12	144

Table 5. (continued)

<u>1948 Groups</u>			<u>1949 Groups</u>		
IQ	d	d ²	IQ	d	d ²
98	-10	100	97	-13	169
98	-10	100	97	-13	169
98	-10	100	95	-15	225
97	-11	121	95	-15	225
97	-11	121	86	-24	576
<u>3774</u>		<u>3672</u>	<u>3840</u>		<u>3568</u>
Mean = 35 $\frac{107.8}{3774.0}$			Mean = 35 $\frac{109.7}{3840.0}$		
Mean = 108			Mean = 110		
Sigma = $\frac{\sqrt{104.9}}{35 \sqrt{3672.0}} = 10.24$			Sigma = $\frac{\sqrt{101.9}}{35 \sqrt{3568.0}} = 10.09$		
Sigma = 10.2			Sigma = 10.1		

Table 6. Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1948 Group

Pupil	IQ	Previous Marks in Mathematics				Av.
		7	8	Grades	9	
1	140	A	A	B	C+	B+
2	128	B+	A	B+	B+	B+
3	128	B+	B	B	B	B
4	124	na*	C	C+	C	C
5	122	C+	B	B	C+	C+
6	121	B	A	B	C+	B
7	120	B+	A	B	C+	B
8	117	D+	C+	D+	C	C
9	117	na	na	A	B+	A
10	116	B	B	A	A	B+
11	115	B	B	C+	C+	C+
12	113	C	B	B	B	C+
13	112	B	B+	B+	A	B+
14	111	B	C	C	C	C
15	111	na	na	B+	B	B
16	110	A	A	B+	B	B+
17	110	C	C	C	C	C
18	110	C	C	D	C	C
19	109	C	C+	C+	C	C+
20	109	D	C	C	C	C
21	108	B-	C	C	C	C
22	107	C	C	C	D	C
23	106	B	B	B	C+	B
24	106	C+	C+	B	B	C+
25	104	D	C	C+	D	D+
26	104	na	na	B	B	B
27	104	B	B-	C	D	C
28	102	na	na	C	B	C+
29	102	B	B	C+	D	C+
30	100	D+	C	C	D	D+

Table 6 (continued)

<u>Previous Marks in Mathematics</u>						
<u>Grades</u>						
Pupil	IQ	7	8	9	10	Av.
31	98	D	C	C	D	D+
32	98	D	C+	C	C	C
33	98	D+	C	D+	D	D+
34	97	D	D	D+	D	D
35	97	na	na	D	D+	D

Mean = 2.43
 Conv. Mean = C+

*na indicates not available

In average letter grades, following conversion
 scale was used:

A = 4.0; B+ = 3.5; B = 3.0; C+ = 2.5;
 C = 2.0; D+ = 1.5; D = 1.0

Table 7. Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1949 Group

<u>Previous Marks in Mathematics</u>						
Pupil	IQ	<u>Grades</u>				Av.
		7	8	9	10	
1	125	B+	A	A	A	A
2	125	na	B+	A	B	B+
3	125	na	B	D	C	C
4	124	A	A	A	B+	A
5	124	B	B+	B+	B	B
6	123	A	C+	B+	B	B
7	122	B	C+	C+	C	C+
8	119	na	na	C+	C+	C+
9	118	C+	C+	C	C	C
10	117	C+	C+	C+	C+	C+
11	115	B	C	B	C	C+
12	113	C	C	B	B	C+
13	112	B+	B	B	C	B
14	112	B	C	C+	C+	C+
15	112	A	B	C+	C+	B
16	112	B	B+	B+	B	B
17	111	C	D+	C+	C+	C
18	110	C	C+	C	C+	C
19	109	B+	A	B	B	B+
20	109	A	C+	C+	B	B
21	108	B	B	C+	C	C+
22	107	D	C	B	C+	C
23	106	B	B	A	B	B
24	106	D	D	D	D	D
25	106	C	C+	D+	E	D+
26	102	na	na	D+	E	D
27	102	C+	C+	C+	D	C
28	100	B	C+	C	D+	C
29	98	na	B	C	C	C+
30	98	C+	C	D+	C	C

Table 1: Summary of Data					
Category	Sub-category	Value 1	Value 2	Value 3	Value 4
Group A	Item 1	10	20	30	40
	Item 2	15	25	35	45
	Item 3	20	30	40	50
	Item 4	25	35	45	55
Group B	Item 1	12	22	32	42
	Item 2	17	27	37	47
	Item 3	22	32	42	52
	Item 4	27	37	47	57
Group C	Item 1	14	24	34	44
	Item 2	19	29	39	49
	Item 3	24	34	44	54
	Item 4	29	39	49	59

Table 7 (continued)

<u>Previous Marks in Mathematics</u>						
Pupil	IQ	<u>Grades</u>				Av.
		7	8	9	10	
31	97	A	B	A	A	B+
32	97	C	C+	D	D+	D+
33	95	na	na	D+	D	D
34	95	B	C	C	B	C+
35	86	C-	C	C+	C+	C+
Mean = 2.471						
Conv. Mean = C+						

na indicates not available

In averaging letter grades, following conversion scale was used:

A = 4.0; B+ = 3.5; B = 3.0; C+ = 2.5;
C = 2.0; D+ = 1.5; D = 1.0

Table 8. Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1948 Group

Pupil	Intel. Quot.	Chron. Age	Problem Solving	Reading Comp.	Spatial Relations
1	140	16-7	11	224	61
2	128	15-10	11	184	44
3	128	16-5	9	171	56
4	124	16-10	9	169	51
5	122	16-5	9	164	58
6	121	16-8	10	186	56
7	120	15-6	5	183	45
8	117	16-1	7	177	42
9	117	16-6	9	173	54
10	116	15-8	7	180	49
11	115	16-6	6	161	38
12	113	16-7	9	200	35
13	112	16-3	4	182	55
14	111	16-4	7	188	44
15	111	15-11	4	160	49
16	110	16-7	9	169	51
17	110	16-3	8	182	46
18	110	15-9	7	146	32
19	109	16-7	6	121	36
20	109	15-11	5	167	47
21	108	15-11	5	181	28
22	107	16-2	7	170	44
23	106	15-11	5	199	53
24	106	15-11	8	163	60
25	104	15-11	8	125	57
26	104	15-8	5	157	40
27	104	16-2	11	126	46
28	102	16-6	5	148	28
29	102	16-1	6	141	37
30	100	15-2	8	143	50

Table 8 (continued)

Pupil	Intel. Quot.	Chron. Age	Problem Solving	Reading Comp.	Spatial Relations
31	98	16-8	7	135	43
32	98	16-6	5	149	47
33	98	15-10	5	156	34
34	97	16-7	4	120	46
35	97	16-3	5	167	43
Range	97-140	(15-2)(16-10)	4-11	120-224	28-61
Median	110	16-3	7	167	46
Mean	108	16-2	7	165	46
Sigma	10.2				

Table 9. Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1949 Group

Pupil	Intel. Quot.	Chron. Age	Problem Solving	Reading Comp.	Spatial Relations
1	125	16-1	10	169	46
2	125	16-7	10	161	50
3	125	15-8	8	195	47
4	124	16-0	12	219	37
5	124	15-10	7	193	45
6	123	17-2	4	200	36
7	122	15-10	7	163	47
8	119	16-0	8	171	40
9	118	16-8	7	161	41
10	117	15-5	5	144	44
11	115	16-5	8	190	39
12	113	15-10	5	178	44
13	112	15-11	6	153	39
14	112	16-5	5	164	48
15	112	16-1	4	170	33
16	112	16-1	7	162	55
17	111	16-5	6	168	30
18	110	16-1	8	168	46
19	109	15-10	7	193	54
20	109	17-6	8	157	35
21	108	16-2	6	232	36
22	107	15-7	8	140	44
23	106	15-10	7	162	39
24	106	16-5	4	125	26
25	106	17-0	4	105	35
26	102	16-4	7	142	39
27	102	15-7	5	178	42
28	100	15-11	8	172	33
29	98	17-0	8	135	43
30	98	16-1	3	130	25

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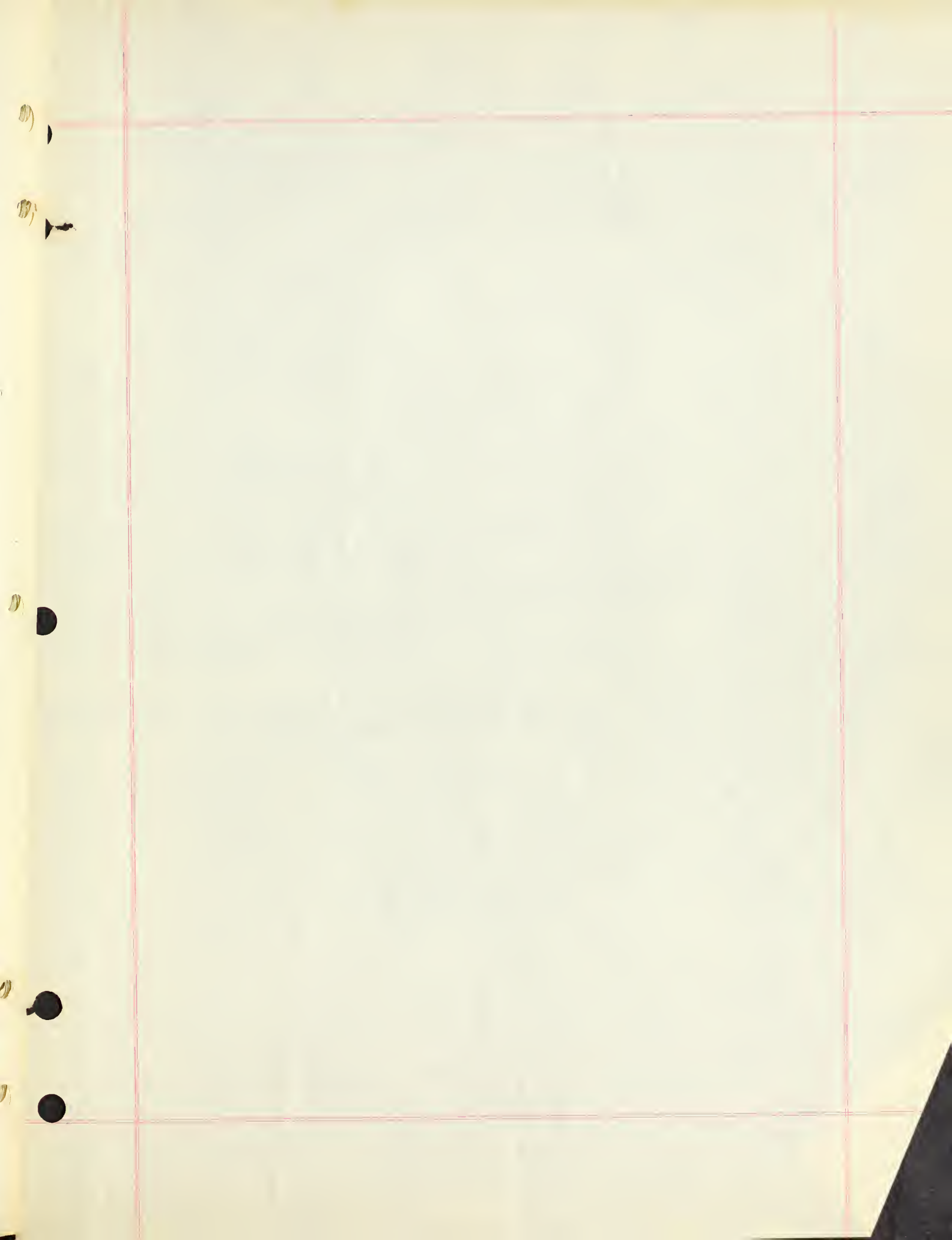
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Table 9 (continued)

Pupil	Intel. Quot.	Chron. Age	Problem Solving	Reading Comp.	Spatial Relations
31	97	16-6	6	162	42
32	97	16-7	4	135	33
33	95	18-4	5	97	27
34	95	15-9	6	141	41
35	86	15-10	4	145	33
Range	86-125	(15-5)(18-4)	3-10	97-232	25-55
Median	110	16-1	7	162	40
Mean	110	16-0	7	165	40
Sigma	10.1				

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